2016 NSW BOS Mathematics Exam Solutions

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Section I

1	2	3	4	5	6	7	8	9	10
B	C	B	A	B	A	A	D	C	D

Q1 $\sin\theta$ is a positive value and $\cos\theta$ is a negative value **B**

Q2 $6\times\frac{1}{30}=\frac{1}{5}$ **C**

Q3 Inverted, turning point at $(2, 3)$ **B**

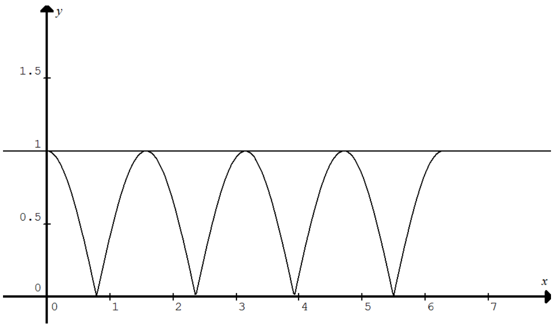
Q4 **A**

Q5 $\frac{d}{dx}\ln(\cos x)=\frac{1}{\cos x}\times(-\sin x)=-\tan x$ **B**

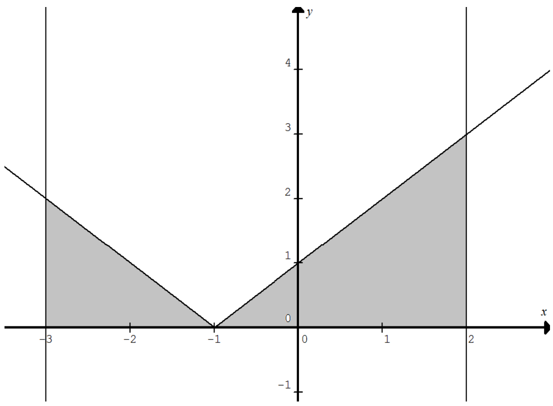
Q6 Period $=\frac{\pi}{3}$ **A**

Q7 Area of shaded region $=\frac{7}{2\pi\times 5}\times\pi\times 5^2=\frac{35}{2}$ **A**

Q8 **D**



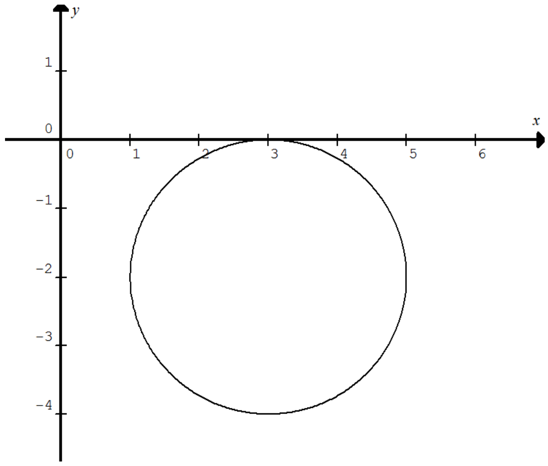
Q9 $\frac{1}{2}(2\times 2+3\times 3)=\frac{13}{2}$ **C**



Q10 $4+\log_2 x=\log_2 2^4+\log_2 x=\log_2(16x)$ **D**

Section II

Q11a



Q11b $\frac{d}{dx}\left(\frac{x+2}{3x-4}\right)=\frac{(3x-4)-3(x+2)}{(3x-4)^2}=\frac{-10}{(3x-4)^2}$

Q11c $|x-2|\le 3$, $\sqrt{(x-2)^2}\le 3$, $(x-2)^2\le 9$, $(x-2)^2-3^2\le 0$, $(x-2+3)(x-2-3)\le 0$, $(x+1)(x-5)\le 0$, $\therefore -1\le x\le 5$

Q11d $\int_0^1 (2x+1)^3\,dx=\left[\frac{(2x+1)^4}{4\times 2}\right]_0^1=\frac{81}{8}-\frac{1}{8}=10$

Q11e $y=-5-4x$, $y=3-2x-x^2$

Let $-5-4x=3-2x-x^2$, $x^2-2x-8=0$, $(x-4)(x+2)=0$

$x=-2, 4$ $\therefore$ $y=3, -21$ respectively

Points of intersection are $(-2, 3)$ and $(4, -21)$.

Q11f $y=\tan x$, $\frac{dy}{dx}=\sec^2 x$

At $x=\frac{\pi}{8}$, gradient $=\frac{dy}{dx}=\sec^2\left(\frac{\pi}{8}\right)=\frac{1}{\left(\cos\frac{\pi}{8}\right)^2}\approx 1.17$

Q11g $\sin\frac{x}{2}=\frac{1}{2}$, $\frac{x}{2}=\frac{\pi}{6},\frac{5\pi}{6}$, $x=\frac{\pi}{3},\frac{5\pi}{3}$

Q12ai $\frac{y-1}{x-6}=\frac{4-1}{2-6}$, $-4(y-1)=3(x-6)$, $-4y+4=3x-18$
$\therefore$ $3x+4y-22=0$

Q12aii Length of $AD=\frac{|3\times 1+4\times 0-22|}{\sqrt{3^2+4^2}}=\frac{19}{5}$

Q12aiii Length of $BC=\sqrt{(1-4)^2+(6-2)^2}=5$
Area $=\frac{1}{2}\times 5\times\frac{19}{5}=\frac{19}{2}$ unit^2

Q12bi $\angle BOA=x°$ (isosceles triangle)
$\angle CBO=2x°$ (external angle equals the sum of interior angles)

Q12bii $\angle BCO=2x°$ (isosceles triangle)
$\angle COB=180°-4x°$ (sum of angles)
$\therefore$ $x°+180°-4x°+87°=180°$ (sum of angles), $\therefore$ $x=\frac{87}{3}=29$

Q12c The cosine rule: $20^2+15^2-2\times 20\times 15\cos(90°-\theta°)=8^2$
$\cos(90°-\theta°)=\frac{20^2+15^2-8^2}{600}=0.935$, $90°-\theta°\approx 21°$, $\theta\approx 69$

Q12di $\frac{dy}{dx}=e^{3x}+3xe^{3x}=e^{3x}(1+3x)$

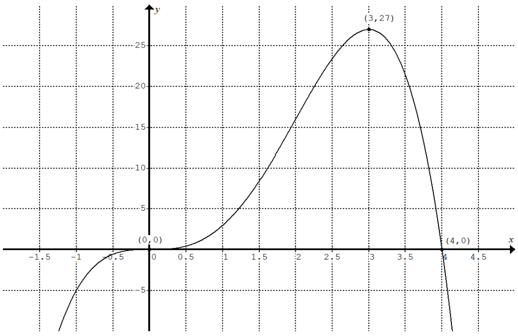
Q12dii
$$\int_0^2 e^{3x}(3+9x)dx=3\times\int_0^2 e^{3x}(1+3x)dx=3\times\left[xe^{3x}\right]_0^2=3\times 2e^6=6e^6$$

Q13ai $y=4x^3-x^4$, $\frac{dy}{dx}=12x^2-4x^3=4x^2(3-x)$
To find the stationary points, let $\frac{dy}{dx}=0$, $\therefore$ $x=0$ and $y=0$ or $x=3$ and $y=27$.

x	<0	0	$0<x<3$	3	>3
dy/dx	positive	0	positive	0	negative
nature		inflection		max. t. p.	

Stationary point $(0, 0)$ is a point of inflection, and stationary point $(3, 27)$ is a local maximum.

Q13aii $y=4x^3-x^4=x^3(4-x)$, x-intercepts: $x=0$, $x=4$



Q13bi $x^2-4x+4=12y+12$, $(x-2)^2=4(3)(y+1)$, focal length $=3$

Q13bii Vertex $(2,-1)$, focus $(2, 2)$

Q13ci $A=10$

Q13cii $5=10e^{-163k}$, $e^{163k}=2$, $k=\frac{\ln(2)}{163}\approx 0.00425$

Q13d Shaded area $=\int_0^1\left(\sqrt{2}\cos\frac{\pi x}{4}-x\right)dx=\left[\frac{4\sqrt{2}}{\pi}\sin\frac{\pi x}{4}-\frac{x^2}{2}\right]_0^1$
$=\frac{4\sqrt{2}}{\pi}\sin\frac{\pi}{4}-\frac{1}{2}=\frac{4}{\pi}-\frac{1}{2}$

Q14ai Approximate increase in area
$=2\times\frac{2-0}{6}[(3-2.5)+4(2.78-2.38)+0]=1.4$ m^2

Q14bi $A_1=0.65\times 100000+5000$
$A_2=0.65A_1+5000=0.65(0.65\times 100000+5000)+5000$

Q14bii $A_2=0.65^2\times 100000+5000+0.65\times 5000$
$A_3=0.65A_2+5000=0.65^3\times 100000+5000+0.65\times 5000+0.65^2\times 5000$
$A_n=0.65^n\times 100000+S_n=0.65^n\times 100000+\frac{5000(1-0.65^n)}{0.35}$

Q14biii $A_{14}=0.65^{14}\times 100000+\frac{5000(1-0.65^{14})}{0.35}\approx 14500$

Q14ci Let w m be the length of the enclosure. $wx=720$, $\therefore$ $w=\frac{720}{x}$
$\therefore$ $\ell=5x+w=5x+\frac{720}{x}$

Q14cii Let $\frac{d\ell}{dx}=0$, $5-\frac{720}{x^2}=0$, $x^2=144$, $x=12$
When $x<12$, $\frac{d\ell}{dx}<0$; when $x>12$, $\frac{d\ell}{dx}>0$, $\therefore$ ℓ is a minimum when $x=12$.
Minimum $\ell=5\times 12+\frac{720}{12}=120$

Q14d $1+x+x^2+x^3+x^4=\frac{1(x^5-1)}{x-1}$
$\therefore$ $\lim_{x\to 1}\frac{x^5-1}{x-1}=\lim_{x\to 1}(1+x+x^2+x^3+x^4)=5$

Q14e $\log 2+\log 4+\log 8+\cdots+\log 512$
$=\log 2+\log 2^2+\log 2^3+\cdots+\log 2^9$
$=\log 2+2\log 2+3\log 2+\cdots+9\log 2=45\log 2$

Q15a For C_1, volume $=\frac{1}{2}\times\frac{4}{3}\pi\times 2^3=\frac{16\pi}{3}$

C_2 is the reflection of C_1 in the *y*-axis and the dilation of C_1 by a factor of $\frac{3}{2}$ in the positive *x*-direction.

For C_2, volume $=\frac{3}{2}\left(\frac{1}{2}\times\frac{4}{3}\pi\times 2^3\right)=8\pi$

Volume of the solid of revolution $=\frac{16\pi}{3}+8\pi=\frac{40\pi}{3}$ unit^3

Q15bi $\Pr(8)+\Pr(8'8)+\Pr(8'8'8)=\frac{1}{8}+\frac{7}{8}\times\frac{1}{8}+\frac{7}{8}\times\frac{7}{8}\times\frac{1}{8}$

$=\frac{1}{8}+\frac{7}{8}\times\frac{1}{8}+\left(\frac{7}{8}\right)^2\times\frac{1}{8}$

Q15bii $\frac{1}{8}+\frac{7}{8}\times\frac{1}{8}+\left(\frac{7}{8}\right)^2\times\frac{1}{8}+\cdots+\left(\frac{7}{8}\right)^{n-2}\times\frac{1}{8}>\frac{3}{4}$

$\frac{\frac{1}{8}\left(1-\left(\frac{7}{8}\right)^{n-1}\right)}{1-\frac{7}{8}}>\frac{3}{4}$, $\quad 1-\left(\frac{7}{8}\right)^{n-1}>\frac{3}{4}$, $\quad \left(\frac{7}{8}\right)^{n-1}<\frac{1}{4}$

$\ln\left(\frac{7}{8}\right)^{n-1}<\ln\frac{1}{4}$, $(n-1)\ln\left(\frac{7}{8}\right)<\ln\frac{1}{4}$, $n-1>\frac{\ln\frac{1}{4}}{\ln\frac{7}{8}}$

.: $n-1>\frac{\ln\frac{1}{4}}{\ln\frac{7}{8}}$, $n-1>10.4$, $n>11.4$

.: smallest n value is 12.

Q15ci $\angle FCB=\angle BAT=90°$

$\angle CFB=\angle ABT$ (alternate angles are equal)

.: the the third pair of corresponding angles are equal

.: ΔFCB and ΔBAT are similar

Q15cii $\angle TSA=\angle AEB=90°$

$\angle TAS+\angle BAE=\angle ABE+\angle BAE=90°$, .: $\angle TAS=\angle ABE$

.: the the third pair of corresponding angles are equal

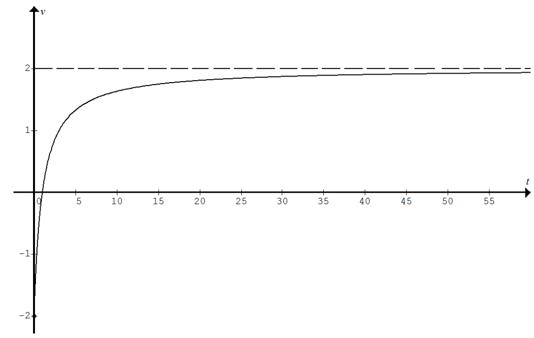
.: ΔTSA and ΔAEB are similar

Q15ciii $\frac{h}{TA}=\frac{y}{AB}$ and $\frac{1}{x}=\frac{TA}{AB}$, .: $h=\frac{yTA}{AB}$, .: $h=\frac{y}{x}$

Q16ai $v=2-\frac{4}{t+1}$, when $t=0$, $v=-2$

Q16aii $a=\frac{dv}{dt}=\frac{4}{(t+1)^2}$. When $v=0$, $t=1$, $a=1$.

Q16aiii As $t\to\infty$, $v\to 2$



Q16aiv Distance $=\left|\int_0^1\left(2-\frac{4}{t+1}\right)dt\right|+\int_1^7\left(2-\frac{4}{t+1}\right)dt$

$=-\left[2t-4\ln(t+1)\right]_0^1+\left[2t-4\ln(t+1)\right]_1^7$

$=-2+4\ln 2+14-4\ln 8-2+4\ln 2$

$=10-4\ln 2$ metres

Q16bi $y=\frac{200}{1+19e^{-0.5t}}$

$\frac{dy}{dt}=-\frac{200}{\left(1+19e^{-0.5t}\right)^2}\times(-0.5)\times 19e^{-0.5t}=\frac{1900e^{-0.5t}}{\left(1+19e^{-0.5t}\right)^2}$

Q16bii When $t=0$, $y=10$, as $t\to\infty$, $y\to 200$, .: $10\le y<200$

Q16biii $y=\frac{200}{1+19e^{-0.5t}}$, $19e^{-0.5t}=\frac{200}{y}-1=\frac{200-y}{y}$

$\frac{dy}{dt}=\frac{1900e^{-0.5t}}{\left(1+19e^{-0.5t}\right)^2}=\frac{100\left(\frac{200-y}{y}\right)}{\left(\frac{200}{y}\right)^2}=\frac{100y(200-y)}{200\times 200}=\frac{y}{400}(200-y)$

Q16biv Rate of growth $=\frac{y}{400}(200-y)$, an inverted parabola with axis of symmetry $y=-\frac{b}{2a}=100$

.: rate of growth is fastest when $y=100$ yabbies

Please inform mathline@itute.com re conceptual and/or mathematical errors.