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2016 Specialist Mathematics

Year 12 Modelling Task

Time allowed: 2.5 hours

You are allowed: 1 bounded reference, 1 CAS, 1 scientific calculator
Working must be shown for questions worth 2 or more marks. Total: 80 marks

Theme: Flight paths

The base (ground level) of a regional airfield control tower is located at the origin O.
Control centre C is inside the control tower and it is 25 m above the ground.
The airfield is horizontal. Unit vectors $\tilde{i}$ and $\tilde{j}$ point to the east and north respectively, and unit vector $\tilde{k}$ points vertically upward.
Distance is in metres and time in seconds.

Helicopter H on the ground takes off in a straight line in the SE direction, climbing at an angle of $30°$ to the horizontal.
Helicopter H takes off at time $t = 0$ and its initial position is $\tilde{r} = 1000\,\tilde{j}$.
H maintains a constant speed of 20 m s^{-1} for 60 seconds. It stops climbing at $t = 60$ s, and flies horizontally in the SE direction at a speed of $10\sqrt{3}$ m s^{-1}.

Question 1

a. Write down the position vector $\tilde{r}_{\text{C}}$ of Control centre C. 1 mark

b. Show that the velocity of H is $\tilde{v} = 5\left(\sqrt{6}\,\tilde{i} - \sqrt{6}\,\tilde{j} + 2\tilde{k}\right)$ in the first 60 s. 4 marks

c. Show that the flight path of H can be modelled by the position vector $\tilde{r}_{\text{H}} = 5\left(\sqrt{6}\,t\,\tilde{i} + \left(200 - \sqrt{6}\,t\right)\tilde{j} + 2t\,\tilde{k}\right)$ in the first 60 s.
3 marks

d. Find the maximum altitude of H above the ground. 1 mark

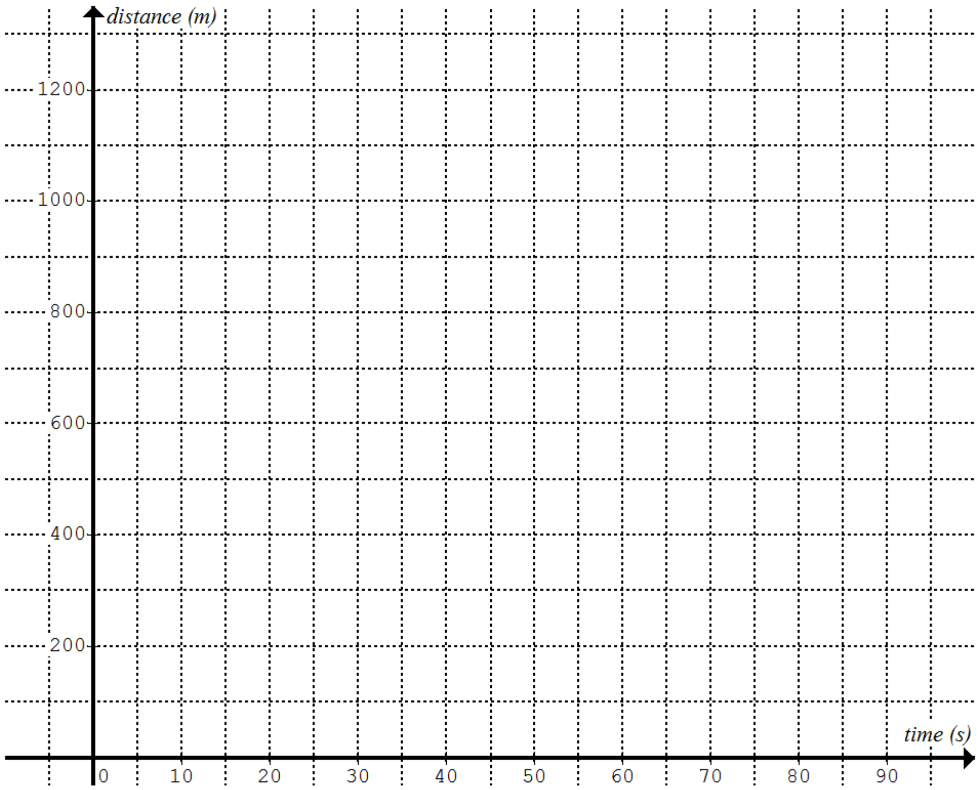
e. Show that the position vector of H for $t > 60$ is $\underset{\sim}{r}_{\text{H}} = 5\left(\sqrt{6}\,t\,\underset{\sim}{i} + \left(200 - \sqrt{6}\,t\right)\underset{\sim}{j} + \alpha\,\underset{\sim}{k}\right)$.
Find the value of α. 3 marks

f. In terms of t, find the distance of H from C in the first 60 s. Simplify your answer. 3 marks

g. In terms of t, find the distance of H from C for $t > 60$. Simplify your answer. 2 marks

h. Sketch the graph of distance of H from C against time t for the interval $0 \le t \le 90$ s. Show and label intercepts, end-points and stationary points.

4 marks



i. Calculate the shortest distance of H from C for $t \geq 0$ and the time that it occurs. 3 marks

j. Use a dot product to find the shortest distance of H from C in the interval $0 \leq t \leq 60$ s. 5 marks

k. Use a dot product to show that the shortest distance of H from C cannot occur at $t > 60$ s. 3 marks

l. The sun is directly above H while H is in flight. The sun casts a shadow of H on the ground. Determine the speed of the shadow of H at time t. 3 marks

m. Show that the true bearing (°T) of H from the control tower at $t > 0$ s is given by $\tan^{-1}\left(1-\frac{100\sqrt{6}}{3t}\right)+90$.

3 marks

n. Show that H will be SE of the control tower eventually. 2 marks

o. If H maintains a constant speed of V m s^{-1} for 60 seconds, stops climbing at $t = 60$ s, and flies in the SE direction at a speed of $\frac{\sqrt{3}}{2}V$ m s^{-1}, find the flight path of H for $t \geq 0$ in the form of a hybrid vector function of t in terms of V.

4 marks

p. Find V when H is closest to C at $t = 20$ s. 4 marks

q. If H maintains a constant speed of V m s^{-1} for T seconds, stops climbing at $t = T$ s, and flies in the SE direction at a speed of $\frac{\sqrt{3}}{2}V$ m s^{-1}, find the flight path of H for $t \geq 0$ in the form of a hybrid vector function of t in terms of V and T.

2 marks

r. Express V in terms of T as an inequality when H is able to level with C at $t \leq T$. 2 marks

s. Express V in terms of T as an inequality when H can never level with C. 1 mark

H is first level with C at $T = 60$ s. The pilot drops a lollypop at $t = 61$. Ignore air resistance.

t. Find the time taken for the lollypop to fall to the ground. 3 marks

u. Find the horizontal distance travelled by the lollypop while falling. 3 marks

The position of aeroplane P at time $t \geq 0$ is modelled by $\tilde{r} = 5\left(2\sqrt{1+(t-1)^2}\,\tilde{i} - 2\sqrt{2}(t-1)\tilde{j} + |t-2|\tilde{k}\right)$.

The aeroplane is first spotted by Control centre C at $t = 0$.

Question 2

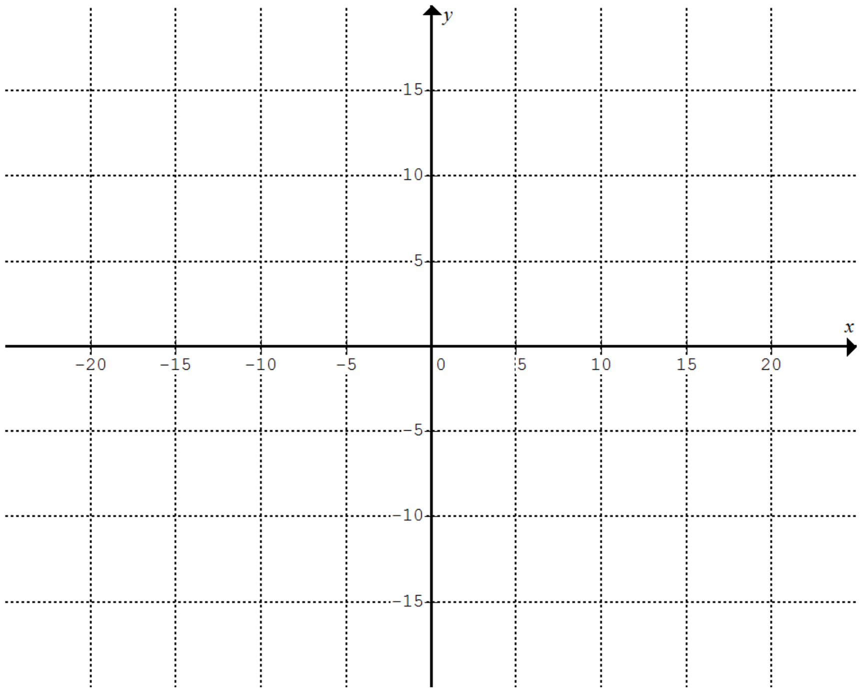
a. Find t when P touches the ground momentarily. 1 mark

b. Find the position of aeroplane P when it touches the ground momentarily. 1 mark

c. Determine the Cartesian equation of the path of the **shadow** cast on the ground when the sun is directly above the aeroplane.

3 marks

d. Sketch the path of the **shadow** of aeroplane P for $t \geq 0$.
Show the coordinates of the endpoint(s), axis-intercept(s) and equation of asymptote(s). 5 marks



e. Find the time when P is closest to C. 2 marks

f. Find the exact speed of P relative to C when it is first spotted. 4 marks

g. At what angle (nearest degree) does the tangent to the flight path of P make with the ground at the time when it is first spotted by C?

3 marks

h. In what direction (nearest °T) does P fly at the time when it is first spotted by C? 2 marks

End of task