2016 VCAA Mathematical Methods Exam 1 Solutions

Q1a $y=\frac{\cos x}{x^2+2}$

$\frac{dy}{dx}=\frac{(x^2+2)(-\sin x)-(\cos x)(2x)}{(x^2+2)^2}=-\frac{\sin x(x^2+2)+2x(\cos x)}{(x^2+2)^2}$

Q1b

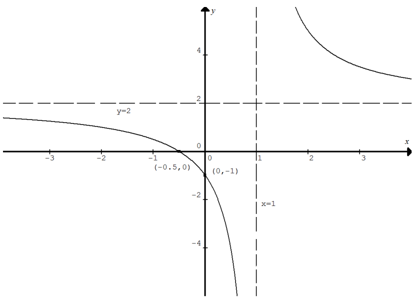
$f(x)=x^2e^{5x}$, $f'(x)=2xe^{5x}+5x^2e^{5x}$, $f'(1)=2e^5+5e^5=7e^5$

Q2a $f(x)=\sqrt{1-2x}$, $f'(x)=\frac{1}{2\sqrt{1-2x}}\times(-2)=-\frac{1}{\sqrt{1-2x}}$

Q2b Gradient of the tangent at $x=-1$:

$m_T=f'(-1)=-\frac{1}{\sqrt{3}}$, $\tan\theta=-\frac{1}{\sqrt{3}}$, $\theta=\frac{5\pi}{6}$ (or 150°)

Q3a



Q3b Area $=\int_2^4\left(2+\frac{3}{x-1}\right)dx=\left[2x+3\log_e(x-1)\right]_2^4$

$=(8+3\log_e 3)-(4+3\log_e 1)=4+3\log_e 3$

Q4a Binomial: $n=4$, $p=\frac{1}{3}$, $x=0$, $\Pr(X=0)=\left(\frac{2}{3}\right)^4=\frac{16}{81}$

Q4b $\Pr(X\ge 1)=1-\Pr(X=0)=1-\frac{16}{81}=\frac{65}{81}$

Q4c Binomial: $n=6$, $p=\frac{16}{81}$, $x=0$, $\Pr(X=0)=\left(\frac{16}{81}\right)^6$

Q5ai $h(x)=f(g(x))=\log_e(x^2+1)$

Q5aii Domain of h is R, range of h is $[0,\infty)$.

Q5aiii $h(x)+h(-x)=2\log_e(x^2+1)$

$f\left((g(x))^2\right)=\log_e(x^2+1)^2=2\log_e(x^2+1)$

.: $h(x)+h(-x)=f\left((g(x))^2\right)$

Q5aiv $h'(x)=\frac{1}{x^2+1}\times 2x$

For stationary points, let $h'(x)=0$, .: $x=0$ and $y=0$

.: only one stationary point $(0,0)$

Since the range of h is $[0,\infty)$, .: $(0,0)$ is an absolute minimum point.

Q5bi Let $y=\log_e(x^2+1)$, $x\in(-\infty,0]$

Equation of the inverse: $x=\log_e(y^2+1)$

.: $y^2=e^x-1$ and its range is $(-\infty,0]$, the domain of k

.: $y=-\sqrt{e^x-1}$, .: $k^{-1}(x)=-\sqrt{e^x-1}$

Q5bii The domain of k^{-1} is the range of k, i.e. $[0,\infty)$.

The range of k^{-1} is $(-\infty,0]$.

Q6a $f(x)=2\sin 2x-1$

Average rate of change of $f=\frac{f\left(\frac{\pi}{6}\right)-f\left(\frac{-\pi}{3}\right)}{\frac{\pi}{6}-\frac{-\pi}{3}}$

$=\frac{\sqrt{3}-1-(-\sqrt{3}-1)}{\frac{\pi}{2}}=\frac{4\sqrt{3}}{\pi}$

Q6b Average value of $f=\frac{1}{\frac{\pi}{2}}\int_{-\frac{\pi}{3}}^{\frac{\pi}{6}}(2\sin 2x-1)dx$

$=\frac{2}{\pi}\left[-\cos 2x-x\right]_{-\frac{\pi}{3}}^{\frac{\pi}{6}}=\frac{2}{\pi}\left(\left(-\frac{1}{2}-\frac{\pi}{6}\right)-\left(\frac{1}{2}+\frac{\pi}{3}\right)\right)$

$=-\frac{2}{\pi}\left(1+\frac{\pi}{2}\right)=-\frac{\pi+2}{\pi}$

Q7a Number of faulty motors in one hour $=0.05\times 40+0.08\times 50=6$

$\Pr(\text{faulty})=\frac{6}{90}=\frac{1}{15}$

Q7b $\Pr(A\mid\text{faulty})=\frac{\Pr(A\cap\text{faulty})}{\Pr(\text{faulty})}=\frac{0.05\times\frac{4}{9}}{\frac{1}{15}}=\frac{1}{3}$

Q8a $\frac{d}{dx}\frac{x^k}{k^2}(k\log_e x-1)=\frac{x^{k-1}}{k}(k\log_e x-1)+\frac{x^k}{k^2}\times\frac{k}{x}$

$=\frac{x^{k-1}}{k}(k\log_e x-1)+\frac{x^{k-1}}{k}=x^{k-1}\log_e x$

.: $\int x^{k-1}\log_e x\,dx=\frac{x^k}{k^2}(k\log_e x-1)$

Q8bi $\Pr\left(X>\frac{1}{e}\right)=\int_{\frac{1}{e}}^{1}-4x\log_e x\,dx=-4\left[\frac{x^2}{4}(2\log_e x-1)\right]_{\frac{1}{e}}^{1}$

$=-4\left[\frac{-1}{4}-\frac{-3}{4e^2}\right]=1-\frac{3}{e^2}$

Q8bii $e>\frac{5}{2}$, .: $e^2>\frac{25}{4}$, $\frac{1}{e^2}<\frac{4}{25}$, $\frac{3}{e^2}<\frac{12}{25}$, $-\frac{3}{e^2}>-\frac{12}{25}$

$1-\frac{3}{e^2}>1-\frac{12}{25}=\frac{13}{25}$, .: $\Pr\left(X>\frac{1}{e}\right)>\frac{1}{2}$, hence median of $X>\frac{1}{e}$.

Please inform mathline@itute.com re conceptual and/or mathematical errors