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2017

Further Mathematics

Trial Examination 1

Core – Data analysis
Recursion and financial modeling

Module 3 – Geometry and measurement

Module 4 – Graphs and relations

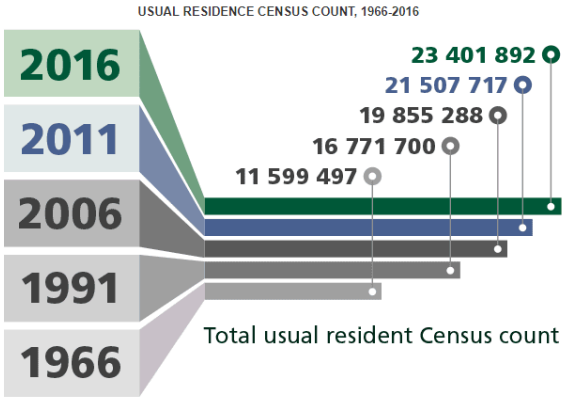
SECTION A – Core

Instructions for Section A

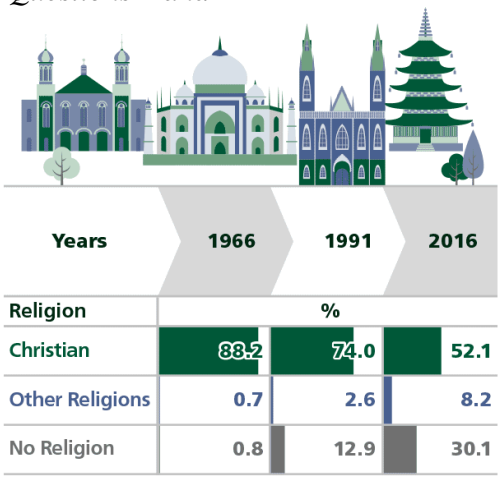
Answer **all** questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if **more than one** answer is completed for any question.
Unless otherwise indicated, the diagrams in this examination are **not** drawn to scale.

Data analysis

The following information relates to Questions 1 and 2



Usual Residence Census counts. Excludes overseas visitors, includes Other Territories.
ABS Source: Census of Population and Housing 1966, 1991, 2006, 2011, 2016



Years	1966	1991	2016
Religion	%		
Christian	88.2	74.0	52.1
Other Religions	0.7	2.6	8.2
No Religion	0.8	12.9	30.1

Other religions includes: Buddhism, Hinduism, Islam, Judaism and other religions.
No Religion includes: Secular Beliefs (e.g. Atheism) and Other Spiritual Beliefs (e.g. New Age).
ABS Source: Census of Population and Housing 1966, 1991, 2016

Question 1

Consider **Total usual resident Census count** in the graphics above as the Australian population.
Which one of the following is a correct statement?

A. Year 1991 had the greatest increase in population.

B. The annual growth rate in population was constant from 1966 to 2016.

C. The population had increased by 100% approximately from 1966 to 2016.

D. The rate of population growth in 2016 was twice of that in 1966.

E. The average population growth per year from 2011 to 2016 was 2 million approximately.

Question 2

Consider the 1966, 1991 and 2016 Census information on religion in Australia.
The figures shown in the table are % of the population.
Which one of the following is a correct statement?

A. The total number of Christian in Australia in 2016 was less than that in 1966.

B. The total number of Christian in Australia in 2016 was 36.1% less than that in 1966.

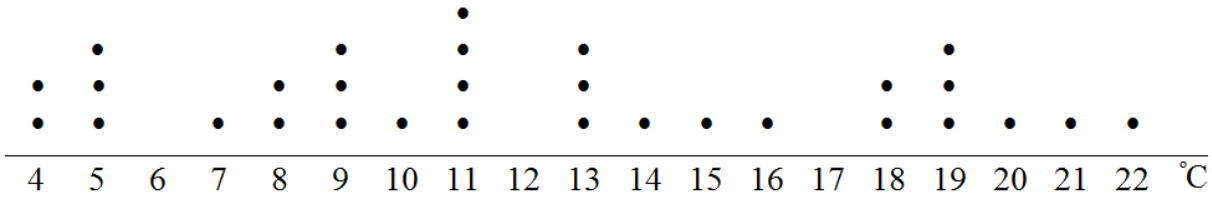
C. The ratio, *population with religion in 2016* : *population with no religion in 2016*, was 1 : 2.

D. The proportion of the population with no religion in 2016 was about 40 times that in 1966.

E. In 2016 about 36.1 % of Christian became non-religious.

The following information relates to Questions 3 and 4

The dot plot below displays the minimum daily temperature (in °C) recorded at a location on each of the 30 days in June 2016.



Question 3

The average minimum daily temperature was closest to

A. 11

B. 11.5

C. 12

D. 12.5

E. 13.5

Question 4

A dot plot is best used to

A. display the distribution of a set of numerical data

B. display the distribution of a categorical variable

C. investigate the association between two categorical variables

D. investigate the association between two numerical variables

E. investigate the association between a categorical variable and a numerical variable

The following information relates to Questions 5, 6 and 7

Question 5

The time (in hours) spent by each of 16 students on the internet on a Friday evening was recorded:

0, 0.1, 0.2, 2.8, 3, 3, 4, 4, 4.2, 4.5, 4.5, 4.5, 4.5, 5.5, 6, 7.1

Which one of the following is true?

A. mode > median > mean

B. mode > mean > median

C. median < mode < mean

D. mean < mode < median

E. mean > median > mode

Question 6

The number of outliers is

A. 0

B. 1

C. 3

D. 4

E. 5

Question 7

The time spent on the internet by some students in the sample was within the interval *sample mean* $\pm$ *one sample standard deviation*.
The percentage of the sample of 16 students is closest to

A. 65

B. 68

C. 69

D. 71

E. 71.2

Question 8

The English and Mathematics examination marks (%) of a class of 22 students are shown in the back-to-back stem plot below.

English							Mathematics					
					8	3	9					
			7	2	2	5	0	8	8			
	8	7	6	4	4	6	5	7	7	8	9	
9	8	7	6	3	1	7	4	6	8	8	9	9
	6	5	5	4	4	8	3	5	5	7	8	
				3	2	9	1	2				

Which one of the following is necessarily true?

A. There is a positive association between a student's English and Mathematics marks.

B. There is a negative association between a student's English and Mathematics marks.

C. The median of English is higher than the median of Mathematics by 1 mark.

D. There is no association between a student's English and Mathematics marks.

E. The percentage of students scoring higher than 50% in English is higher than the percentage of students scoring higher than 50% in Mathematics.

Question 9

The length of a fish, in cm, and its age, in months, are recorded monthly over a 12 month period.
A log transformation is used to linearise the relationship between the two variables. The least squares regression line fitted to the transformed data has the following equation.

$$length = 5.0 + 25\log_{10}(age)$$

By extrapolation, the length of the fish (in cm) after 13 months is closest to

A. 30

B. 33

C. 36

D. 38

E. 70

Question 10

For a set of bivariate data that involves explanatory variable t and response variable x, the least squares regression line is $x = 7.9 - 0.55t$ with $r = -0.80$.

The value of the ratio $\frac{s_t}{s_x}$ is closest to

A. -1.5

B. 1.5

C. -0.7

D. 0.7

E. -0.1

Question 11

The following table displays a set of bivariate data.

x	0.44	0.50	0.53	0.59	0.63	0.72	0.83	0.91	1.00
y	1.30	1.00	0.90	0.70	0.60	0.40	0.20	0.10	0.00

The best transformation of one of the variables to linearise the set of data is

A. $\frac{1}{y}$

B. $\frac{1}{x}$

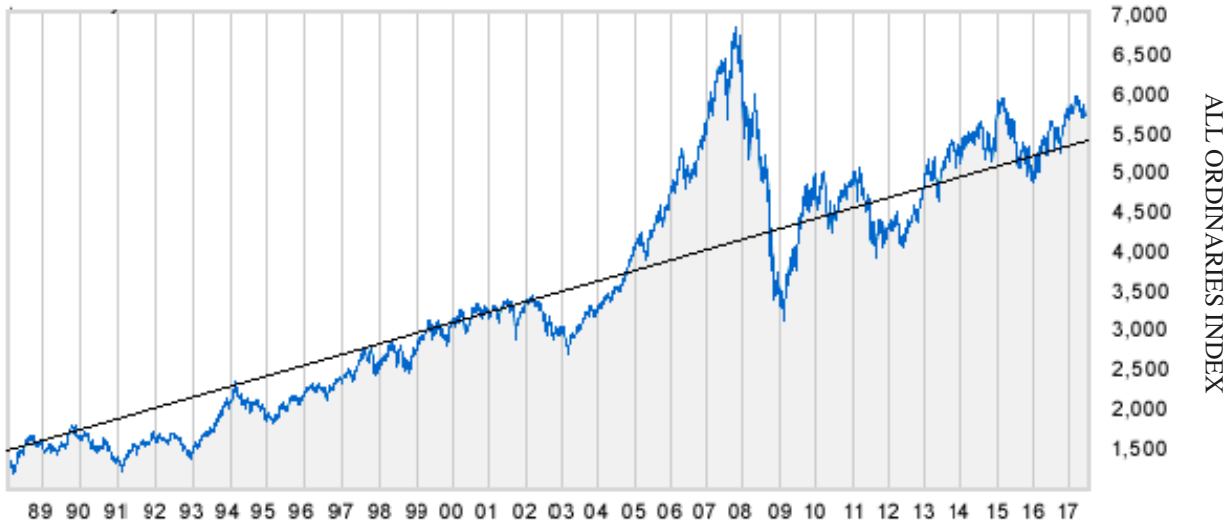
C. x^2

D. $\log_{10} x$

E. $\log_{10} y$

The following information relates to Questions 12 and 13

The following time series shows the Australian All Ordinaries Indices from 1988 to 2017.
It is made up of the share prices for 500 of the largest companies listed on the Australian Securities Exchange.
The All Ordinaries was first established in January 1980, and the index was 500.
A regression line is fitted to the time series.
Note: Shares are traded daily except on weekends and some public holidays. An index is determined at the end of a trading day.



(SOURCE: THE SYDNEY MORNING HERALD)

Question 12

Which one of the following does **not** feature in the above time series?

A. Long-term direction

B. Irregular short-term fluctuations

C. Seasonality

D. One-off real world event

E. Signs of discontinuity

Question 13

Let January 1980 be year 0, January 1990 be year 10 etc.
The equation of the regression line fitted to the time series is best given by

A. $index = 100 + 149.3 \times year$

B. $year = -7.4 + 0.0074 \times index$

C. $index = 500 + 143 \times year$

D. $year = -3.5 + 0.0070 \times index$

E. $index = 1000 + 135 \times year$

The following information relates to Questions 14, 15 and 16

The following incomplete table (missing entries in the shaded cells) shows the long-term average rainfall (in mm) for two of the four seasons. Also shown are two of the four seasonal indices.

	Summer	**Autumn**	**Winter**	**Spring**
Long-term average rainfall	65.2	68.5		
Seasonal index		1.02		1.16

Question 14

The seasonal index for summer is closest to

A. 0.93

B. 0.95

C. 0.97

D. 0.99

E. 1.01

Question 15

The long-term average rainfall (in mm) for winter is closest to

A. 57.1

B. 58.2

C. 59.3

D. 60.4

E. 61.5

Question 16

In 2016, the rainfall in spring was 71.3 mm.
The deseasonalised rainfall (in mm) for spring is closest to

A. 57.1

B. 58.2

C. 59.3

D. 60.4

E. 61.5

Recursion and financial modelling

Question 17

Given $T_3 = 3$ and $T_{n+1} = \frac{T_n}{2} - 1$, the first term T_0 is

A. 38

B. 34

C. 18

D. 14

E. 8

Question 18

A worker agrees on a salary rise of fixed 3.5% p.a. for the next five years. The starting salary is $\$60000$.
A recurrence relation that can be used to determine the salary after n years, S_n (in dollars), is

A. $S_{n+1} = 0.035 \times S_n,\ S_0 = 60000,\ n \le 5$

B. $S_{n+1} = 0.965 \times S_n,\ S_0 = 60000,\ n \le 5$

C. $S_{n+1} = 0.035 \times S_n + 60000,\ S_0 = 60000,\ n \le 4$

D. $S_{n+1} = 1.035 \times S_n,\ S_0 = 60000,\ n \le 4$

E. $S_{n+1} = 3.5 \times S_n,\ S_0 = 60000,\ n \le 4$

Question 19

Consider the recurrence relation: $A_0 = 160000,\ A_{n+1} = 1.005(A_n + 1000)$
This relation could be used to model

A. a reducing balance depreciation of an asset initially valued at $\$160000$.

B. a reducing balance loan with periodic repayment of $\$1000$.

C. an annuity investment with $\$1000$ added to the account at the end of each period.

D. an annuity investment with $\$1005$ added to the account at the end of each period.

E. an annuity investment with $\$1005$ added to the account at the beginning of each period.

Question 20

The following amortization table for a reducing balance home loan shows the amounts after the first two payments to repay the loan.

Payment number	Payment $	Interest $	Principal reduction $	Balance of loan $
0	0	0.00	0.00	300000.00
1	1800.00	1200.00	600.00	299400.00
2	1800.00	1197.60	602.40	298797.60

The balance of the loan after payment number 3 is closest to

A. $298195

B. $298194

C. $298193

D. $298192

E. $298191

Question 21

An annuity can be modelled by the recurrence relation: $A_0 = 80000,\ A_{n+1} = x \times A_n - 4000$
Which of the following statements about the value of x in the relation is true for the annuity to last indefinitely?

A. $x \leq 0.95$

B. $x < 1$

C. $x > 1$

D. $x > 1.02$

E. $x \geq 1.05$

Question 22

The value of an annuity, $\$A_n$, after n monthly payments of $\$Q$ have been made, can be determined using the recurrence relation

$$A_0 = 250000,\ A_{n+1} = 1.005 \times A_n - Q$$

If $\$248095$ is the value of the annuity after two payments have been made, the value of Q is closest to

A. 1125

B. 1150

C. 1200

D. 1600

E. 2200

Question 23

An investor wishes to double her investment in 10 years.
The fixed interest rate is $r\%$ p.a. compounding monthly.
The value of r is closest to

A. 7.18

B. 7.17

C. 6.96

D. 6.95

E. 6.74

Question 24

A housing loan of $\$447\,000$ at 5.2 % p.a. compounding monthly requires a monthly payment of $3000 to repay the loan.
If the interest rate is 6.0 % p.a. the number of **extra** monthly payments of $3000 required to repay the loan is closest to

A. 35

B. 32

C. 29

D. 26

E. 23

SECTION B – Modules

Instructions for Section B

Answer **all** questions
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
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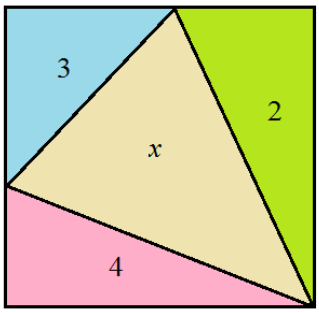
Module 3 – Geometry and measurement

Question 1

Four triangles form a square as shown in the diagram below. 2, 3, 4 and x are the areas of the triangles.



The value of x is

A. 5

B. 6

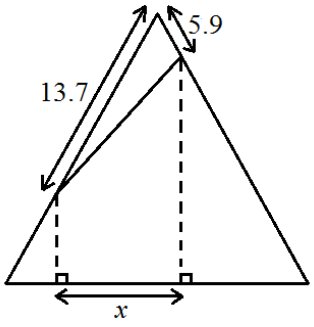
C. 7

D. 8

E. 16

Question 2

A line segment touches two sides of an **equilateral** triangle.
13.7 , 5.9 and x are length measurements as shown in the diagram below.



The value of x is

A. 9.8

B. 8.6

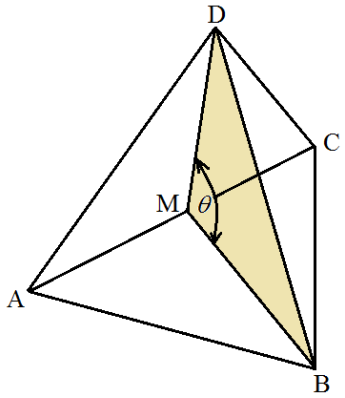
C. 7.8

D. 7.6

E. 7.1

Question 3

In the diagram below, ABCD is a regular tetrahedron (also known as triangular pyramid).
The four surfaces ABC, ABD, BCD and CAD are **equilateral** triangles.
M is the midpoint of AC. The measure of $\angle BMD$ is θ.



The value of θ is

A. $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

B. $\cos^{-1}\left(\frac{1}{3}\right)$

C. $\sin^{-1}\left(\frac{1}{2}\right)$

D. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

E. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Question 4

A boat at point P moves in the direction 320°T for 2.5 km to Island A, and then 230°T for 3.5 km to Island B.
The bearing of point P from Island B is closest to

A. 266°T

B. 086°T

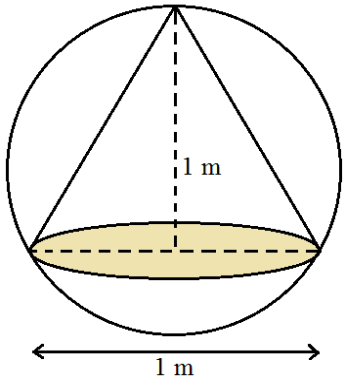
C. 281°T

D. 054°T

E. 324°T

Question 5

A cone of height 1 m and diameter 1 m just fits inside a sphere of radius r metres.



The value of r is

A. 0.600

B. 0.625

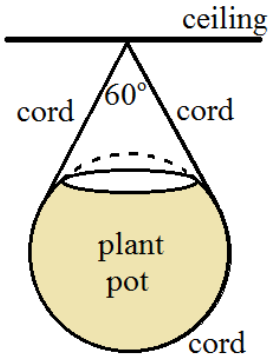
C. 0.650

D. 0.675

E. 0.700

Question 6

A **spherical** plant pot of radius 12 cm is suspended by cords.
One end of the longest cord is fastened to the ceiling, goes around the plant pot and touches its lowest point. The other end of the cord is fastened to the ceiling at the same point as the other end. The two ends of the cord make an angle of $60°$. Refer to the diagram below.



The total length (in cm) of the cord is closest to

A. 72

B. 82

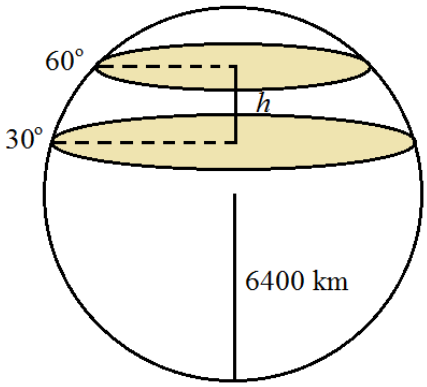
C. 92

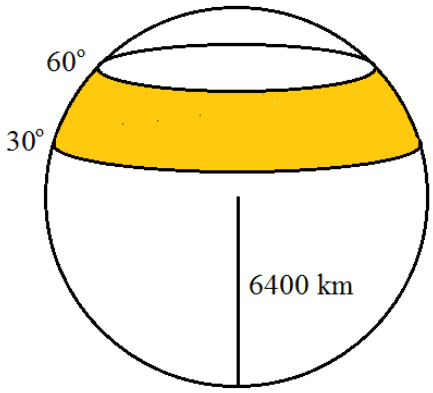
D. 102

E. 112

The following information relates to Questions 7 and 8

The diagram below left shows two parallel circular planes across Earth. One plane is at latitude $30°$, the other at latitude $60°$. The two planes are separated by h km. The radius of Earth is 6400 km.
The diagram below right shows a ring band around the surface of Earth between latitudes $30°$ and $60°$.





Question 7

The value of h (in km) is closest to

A. 2040

B. 2140

C. 2240

D. 2340

E. 2440

Question 8

The ring band around the surface of Earth between latitudes $30°$ and $60°$ has an area of 94200000 km^2.
The area (in km^2) of the larger section of the ring band between longitude $12°$ east and $15°$ west of the Greenwich meridian is closest to

A. 785000

B. 7070000

C. 78500000

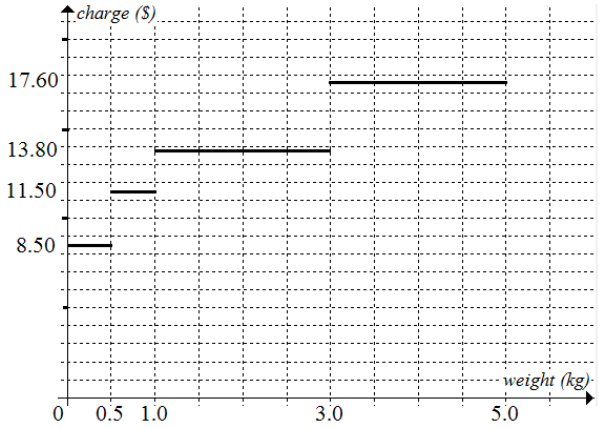
D. 87100000

E. 93400000

Module 4 – Graphs and relations

Question 1

Australian Post has four sizes of sachet bag for parcel delivery. The sizes are (up to) 0.5 kg, 1.0 kg, 3.0 kg and 5.0 kg. The corresponding prices are shown in the graph below.



Some loose items (total weight is 3.1 kg) are to be sent through Australian Post using sachet bags.
The cheapest way is to put the items in x bags. The value of x is

A. 1

B. 2

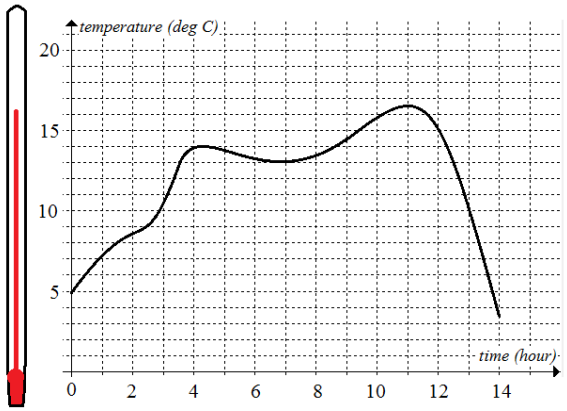
C. 3

D. 4

E. 5

Question 2

The tip of the alcohol inside the thermometer moves up and down the scale as the temperature changes.



The number of degrees moved up and down the scale by the tip of the alcohol column in the first **12** hours is closest to

A. 11.5

B. 14.5

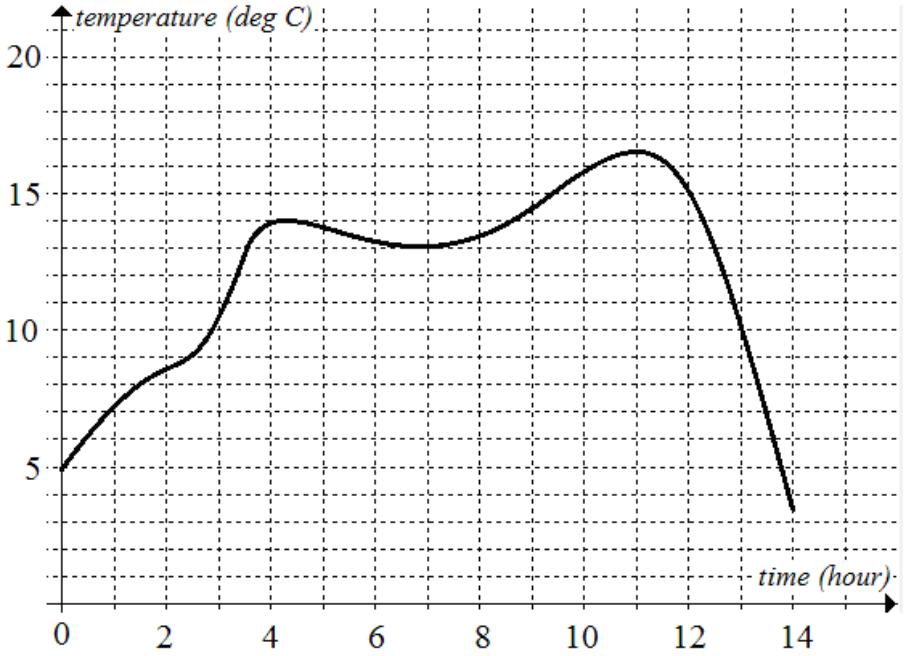
C. 24.5

D. 26.5

E. 28.5

The following information relates to Questions 3 and 4

Consider the change in temperature shown in the following graph.



Question 3

The average rate of change in temperature over the first 12 hour period (in $°C\,h^{-1}$) is closest to

A. 0.83

B. 0.93

C. 1.03

D. 1.13

E. 1.23

Question 4

The number of hours that the temperature was over $13.5°C$ is closest to

A. 5.1

B. 5.5

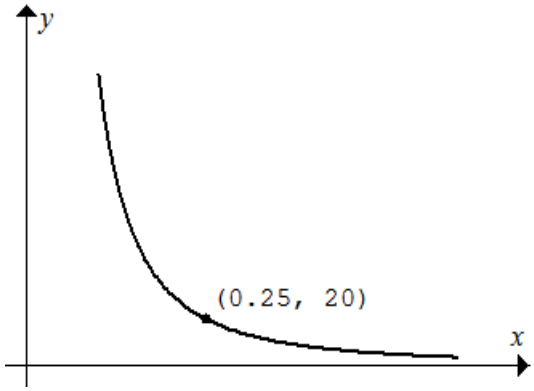
C. 5.9

D. 6.3

E. 6.7

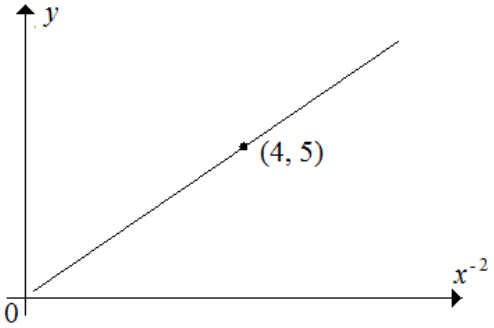
Question 5

The point $(0.25, 20)$ lies on the graph of $y = k\,x^n$ as shown below.

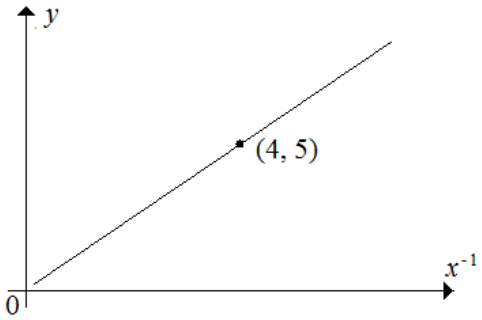


Another graph that represents this relationship between y and x could be

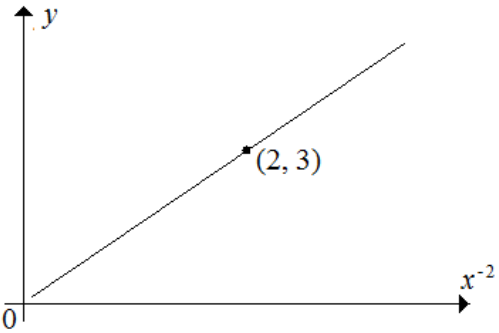
A.



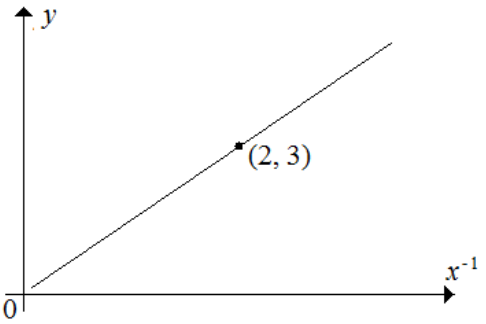
B.



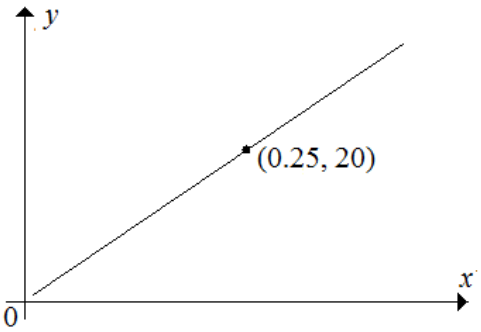
C.



D.

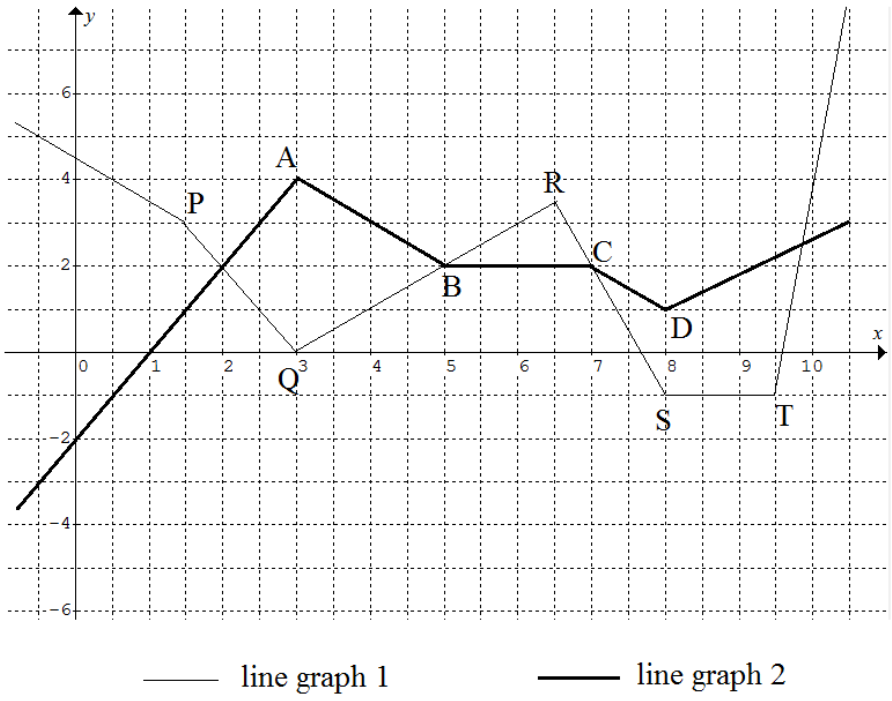


E.



Question 6

Line graph 1 has five corner points P, Q, R, S and T.
Line graph 2 has four corner points A, B, C and D. See diagram below.
A third line graph (not shown in the diagram) is formed when line graph 1 and line graph 2 are **added**.



The total number of corner points in the third line graph (not shown) in the interval $0 < x < 10$ is

A. 5

B. 6

C. 7

D. 8

E. 9

Question 7

A worker drinks both tea and coffee in no particular order throughout the day.
Let c be the number of cups of coffee, and t the number of cups of tea consumed by the worker in a day.
The worker has no more than 4 cups of tea for every 3 cups of coffee consumed.
An inequality that represents this situation is

A. $c \le t + 1$

B. $t \le c + 1$

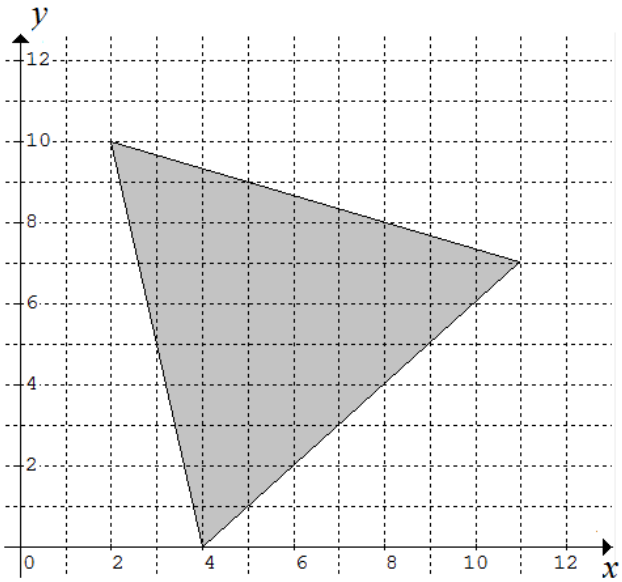
C. $c + t \le 7$

D. $4t \le 3c$

E. $3t \le 4c$

Question 8

Consider the shaded region bounded by three line segments in the diagram below.



Which one of the following **cannot** be true for **some** points (x, y) in the region?

A. $y < 9$

B. $x \leq 11$

C. $x - y > 4$

D. $x + 3y \geq 32$

E. $5x + y > 20$

End of Exam 1