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2017 Further Mathematics

Year 12 Problem Solving Task

Time allowed: 2 hours plus

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Theme: Daily dietary requirements

Required skills/knowledge: Linear equations, inequalities, graphs and linear programming

The task: Based on the given information, a set of constraints is formulated as inequalities and the objective function is developed as an equation. The constraints are analysed and graphed, the feasible region for the objective function is defined and studied, and the optimal value of the objective function is determined.

Further constraints are introduced. Their effects on the feasible region and the optimisation of the objective function are determined.

Two parameters are included in the information. They are varied one at a time, and the effects on the feasible region/objective function are investigated. The values of each parameter are to be selected by students.

Start of task:

A dietitian recommends that the minimum daily requirements of proteins, vitamins and carbohydrates are 30, 12 and 18 units respectively. The three nutrients can be obtained by consuming meat and vegetables. The nutrient contents (units) of meat (per kg) and vegetables (per kg) are shown in the following table. There are two parameters, a and b, in the table. Their values depend on the types of vegetables and meat in the diet.

	Meat	**Vegetables**
Proteins	24	a
Vitamins	12	12
Carbohydrates	b	36

Let x kg of meat and y kg of vegetables be the daily amount used in the diet.
The cost of meat is stable at \$10 per kg, whilst the cost of vegetables is seasonal at $\$n$ per kg.

Part I

Question 1a
Write down an inequality in terms of x and y for the daily dietary intake of vitamins.

Question 1b
Write down an inequality in terms of a, x and y for the daily dietary intake of proteins.

Question 1c
Write down an inequality in terms of b, x and y for the daily dietary intake of carbohydrates.

Let $\$C$ be the daily total cost of meat and vegetables for the diet.

Question 2
Write down an equation for C in terms of n, x and y.

Let $n = 5$, $a = 36$ and $b = 12$ **for Question 3 to Question 12 only**.

Question 3
Fill in the blank cells with the values of C for the corresponding values of x and y.

x \ y	0.00	0.25	0.50	0.75	1.00
0.00					
0.25					
0.50					
0.75					
1.00					

Question 4
Cross out those entries in the completed table, which do not satisfy the minimum daily requirements of proteins, vitamins and carbohydrates.

Question 5
Among the entries in the table, what is the lowest cost which satisfies the minimum daily requirements of proteins, vitamins and carbohydrates?

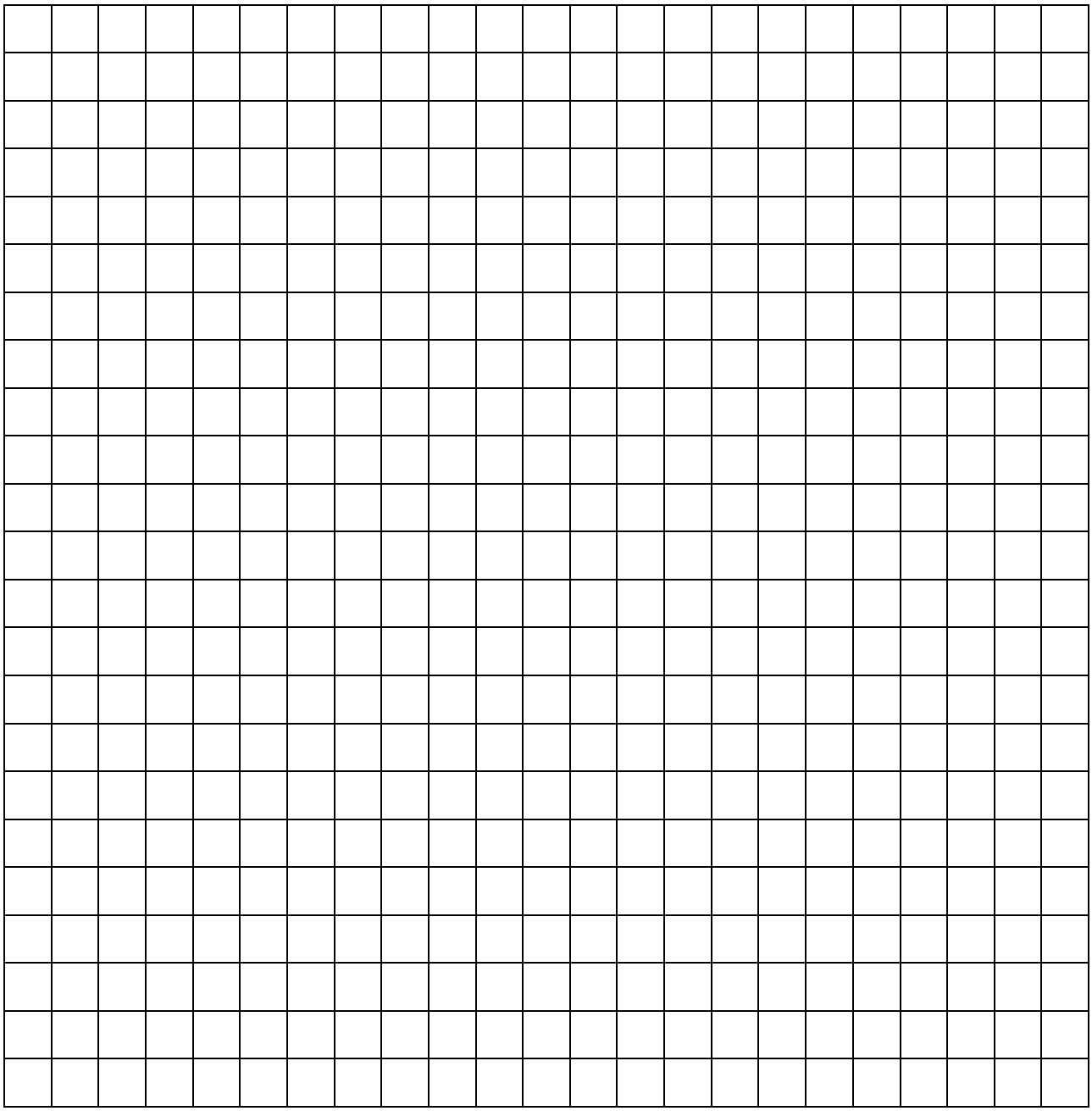
Question 6
Among the entries in the table, find the non-zero x and y values for the lowest cost diet which satisfies the minimum daily requirements of proteins, vitamins and carbohydrates.

Question 7
What is the lowest cost of a meat only diet which satisfies the minimum daily requirements of proteins, vitamins and carbohydrates? (Note: Not an entry in the table)

Tabulating C as shown above is a method in finding the lowest cost of a satisfactory diet for **discrete** values of x and y. Graphical method is more suitable when the values of x and y are **continuous**.

Question 8

On the grid diagram below show the graph of the minimum daily requirements of proteins, vitamins and carbohydrates by shading the region **NOT** satisfying the requirements. The horizontal and vertical axes are for x and y respectively. Make full use of the diagram showing $x \leq 2.0$ and $y \leq 2.0$.



Question 9

On the above grid diagram sketch the graph of the objective function for $C = 9.50$.

Question 10
Determine the lowest cost which satisfies the minimum daily requirements of proteins, vitamins and carbohydrates.

Question 11
If the dieter wishes to have at least 0.25 kg of meat, determine the lowest cost which satisfies the minimum daily requirements of proteins, vitamins and carbohydrates.

Question 12
If the dieter wishes to have no more than 0.50 kg of vegetables, determine the lowest cost which satisfies the minimum daily requirements of proteins, vitamins and carbohydrates.

Part II

Investigation 1

Let the values of a and b be the same as in **Part I**, i.e. $a = 36$ and $b = 12$.
Now the cost of vegetables has jumped from $n = 5$ to $n = 10$.

Question 13a
Determine the lowest cost which satisfies the minimum daily requirements of proteins, vitamins and carbohydrates.

Question 13b
Find the range (kg) of daily meat consumption which keeps the lowest cost determined in Question 13a and satisfies the minimum daily requirements of proteins, vitamins and carbohydrates.

Question 14
If the cost of vegetables had jumped to $n = 14$ instead, determine the lowest cost which satisfies the minimum daily requirements of proteins, vitamins and carbohydrates.

Investigation 2

Let the values of n and b be the same as in **Part I**, i.e. $n = 5$ and $b = 12$.
Investigate the effect on the lowest cost which satisfies the minimum daily requirements of proteins, vitamins and carbohydrates when the value of a increases and decreases from 36.

Investigation 3

Let the value of a be the same as in **Part I**, i.e. $a = 36$.
The cost of vegetables has jumped from $n = 5$ to $n = 14$.
Investigate the effect on the lowest cost which satisfies the minimum daily requirements of proteins, vitamins and carbohydrates when the value of b increases and decreases from 12.

End of Problem Solving Task