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2017 Mathematical Methods Trial Exam 2 Solutions

SECTION A – Multiple-choice questions

1	2	3	4	5	6	7	8	9	10
E	D	C	E	A	D	B	A	A	B

11	12	13	14	15	16	17	18	19	20
D	C	E	D	E	A	A	B	A	B

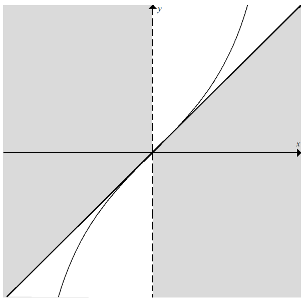
Q1 $y=\left(a\left(\frac{b+4}{a}\right)-b\right)^{1.5}=8$ **E**

Q2 $x-1\geq 0$ and $2+x>0$, .: $x\geq 1$ **D**

Q3 $g(x)=0.2f(0.1x+0.2)+0.1$
$g(10x-2)=0.2f(0.1(10x-2)+0.2)+0.1=0.2f(x)+0.1$
$h(x)=5g(10x-2)-0.5=f(x)$ **C**

Q4 The inverse is $x=e^{\left(\frac{y}{10}\right)}-\log_e\left(\frac{y}{10}\right)$ **E**

Q5 $y=mx$ lies in the shaded region, .: $m\leq 1$ **A**



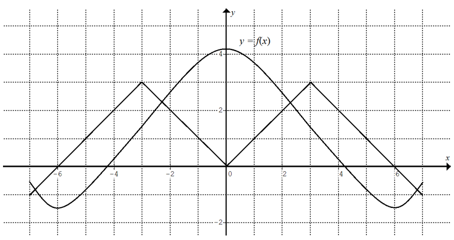
Q6 $\cos\left(\frac{\pi(x-1)}{2}\right)=\sin\left(\frac{\pi x}{2}\right)$, $\cos\left(\frac{\pi(x+1)}{2}\right)=-\sin\left(\frac{\pi x}{2}\right)$ **D**

Q7 $(x-p)^3=x^3-3px^2+3p^2x+p^3$, .: $a=-3p$, $b=3p^2$
.: $a^2=3b$ **B**

Q8 Try $x=0$, $by+z=1$, $dy+z=0$, eliminate z, $y=\frac{1}{b-d}$ and
$z=-\frac{d}{b-d}=\frac{d}{d-b}$ **A**

Q9 $\frac{15-6}{12}=0.75$ **A**

Q10 The graph shows that $y=f(x)$ is an even function.
.: $f(-x)=f(x)$, $f(x)-f(-x)=0$. **B**



Q11 $e^{2x}=e^x+c$, $(e^x)^2-e^x-c=0$, $e^x=\frac{1\pm\sqrt{1+4c}}{2}$

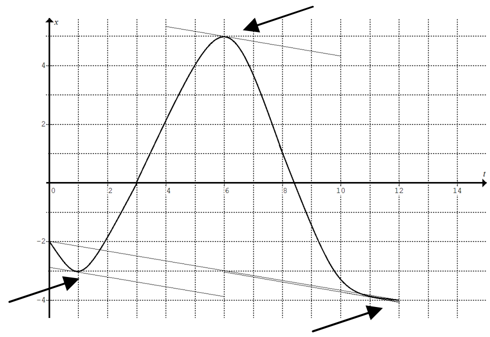
Two distinct roots if $1+4c>0$ and $\frac{1-\sqrt{1+4c}}{2}>0$

.: $-\frac{1}{4}<c<0$ **D**

Q12 The curve $y=a\sin(x)\cos\left(\frac{x}{2}\right)$ including the point at $x=\frac{1}{2}$ is dilated by the same factor in both x and y directions, .: same gradient at $x=\frac{1}{4}$. **C**

Q13 $\left[\frac{e^{bx}}{b}-\frac{2e^{\frac{bx}{2}}}{b}\right]_0^a=\frac{1}{b}$, $e^{\frac{ab}{2}}\left(e^{\frac{ab}{2}}-2\right)=0$, $ab=\log_e 4$ **E**

Q14 **D**



Q15 Number of squares bounded by the graph and the t-axis ≈ 32
Each square represents 1 metre distance.

Average speed $\approx\frac{32}{12}\approx 2.7$ **E**

Q16 $\frac{\Pr(5.5<X<11)}{\Pr(X>5.5)}=\frac{0.6}{0.65}\approx 0.92$ **A**

Q17 $\int_0^{2\pi}\frac{a}{2}(1-\cos x)dx=1$, .: $a=\frac{1}{\pi}$
$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\frac{1}{2\pi}(1-\cos x)dx=\frac{\pi+2}{2\pi}$ **A**

Q18 Choice B is not a random variable, it is a subset of the sample space. **B**

Q19 $n=4$, $p=0.1$ and $q=0.9$
.: $\mu=np=0.4$ and $\sigma=\sqrt{npq}=0.6$ **A**

Q20 The mean of $\hat{P}=p=0.60$

The standard deviation of $\hat{P}=\sqrt{\frac{p(1-p)}{n}}=0.04$

$\Pr(\hat{P}<0.56)\approx 0.16$ (normal approx)
Number of samples $\approx 20\times 0.16\approx 3.2$ **B**

SECTION B

Q1a $y = x(2a - x)$, $p = \frac{2a+0}{2} = a$, $q = 2a^2 - a^2 = a^2$

Q1bi $y = mx + c$ is the equation of the tangent.

Let $mx + c = 2ax - x^2$, .: $x^2 + (m - 2a)x + c = 0$

.: $(m - 2a)^2 - 4c = 0$

Q1bii From part bi, $(m - 2a)^2 = 4c$, $m - 2a = \pm 2\sqrt{c}$

.: $m = 2\left(a \pm \sqrt{c}\right)$

Given $c > q$, i.e. $c > a^2$, .: $\sqrt{c} > a$, $a - \sqrt{c} < 0$

.: $m = 2\left(a - \sqrt{c}\right)$ for negative m.

Q1ci When $a = 1$, $m = 2\left(1 - \sqrt{c}\right)$, $y = 2\left(1 - \sqrt{c}\right)x + c$

.: $2\left(\sqrt{c} - 1\right)x + y = c$

Q1cii Let $y = 0$ for x-intercept, .: $x = \frac{c}{2\left(\sqrt{c} - 1\right)}$

.: $B = \left(\frac{c}{2\left(\sqrt{c} - 1\right)}, 0\right)$

Q1d Area of $\Delta AOB = \frac{1}{2} \times \frac{c}{2\left(\sqrt{c} - 1\right)} \times c = \frac{c^2}{4\left(\sqrt{c} - 1\right)} = \frac{c^2\left(\sqrt{c} + 1\right)}{4(c - 1)}$

Q1e Let $\frac{d}{dc} \frac{c^2}{4\left(\sqrt{c} - 1\right)} = 0$, $c = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$

Min. area of $\Delta AOB = \frac{c^2}{4\left(\sqrt{c} - 1\right)} = \frac{\left(\frac{4}{3}\right)^4}{\frac{4}{3}} = \left(\frac{4}{3}\right)^3$

Q1f Area bounded by the parabola and the x-axis:

$\int_0^2 \left(2x - x^2\right) dx = \frac{4}{3}$

.: min. shaded area $= \left(\frac{4}{3}\right)^3 - \frac{4}{3} = \frac{28}{27}$

Q2a $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Q2b Same area as the region bounded by $y = e^x$, $y = -e^{-x}$ and $x = \pm 1$.

$\int_{-1}^{1} \left(e^x - \left(-e^{-x}\right)\right) dx = 2e$

Q2c $g'(x) = -\frac{1}{x}$ for $x < 0$

Q2d $Q(-\alpha, -\log_e \alpha)$

Q2e $\left(\overline{PQ}\right)^2 = (\alpha - (-\alpha))^2 + (\log_e \alpha - (-\log_e \alpha))^2 = 4\left(\alpha^2 + (\log_e \alpha)^2\right)$

.: $\overline{PQ} = 2\sqrt{\alpha^2 + (\log_e \alpha)^2}$

Q2f Let $\frac{d}{d\alpha} 2\sqrt{\alpha^2 + (\log_e \alpha)^2} = 0$

$\frac{1}{\sqrt{\alpha^2 + (\log_e \alpha)^2}} \times \left(2\alpha + \frac{2\log_e \alpha}{\alpha}\right) = 0$

.: $2\alpha + \frac{2\log_e \alpha}{\alpha} = 0$, .: $\alpha^2 + \log_e \alpha = 0$

Q2g $\alpha \approx 0.653$, $\overline{PQ} \approx 1.55954 \approx 1.560$

Q2h Since $\alpha^2 + \log_e \alpha = 0$, .: $\frac{\log_e \alpha}{\alpha^2} = -1$

Gradient of $\overline{PQ} = \frac{\log_e \alpha}{\alpha}$, gradient of both tangents $= \frac{1}{\alpha}$

.: gradient of $\overline{PQ} \times$ gradient of each tangent $= \frac{\log_e \alpha}{\alpha^2} = -1$

.: PQ is a common normal to both curves.

Q2i Area of the largest square $\approx 1.55954^2 \approx 2.432$

Q3a

$f(x) = 0.0012833\ x^2(x^2 - 8\pi^2) = 0.0012833\ x^2\left(x^2 - \left(2\sqrt{2}\pi\right)^2\right)$

$= 0.0012833\ x^2\left(x + 2\sqrt{2}\pi\right)\left(x - 2\sqrt{2}\pi\right)$

.: $a = 0.0012833$ and $b = c = 2\sqrt{2}\pi$

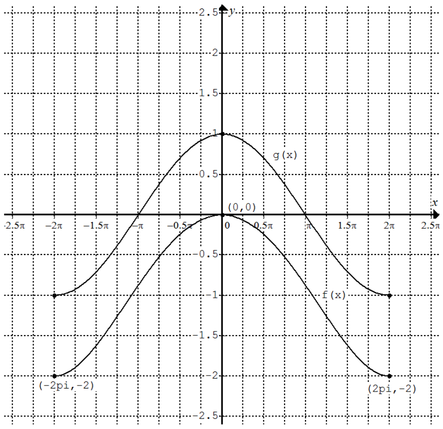
Q3b $f'(x) = 0.0051332\ x^3 - 0.0205328\ \pi^2 x$

Let $f'(x) = 0$, $0.0051332\ x^3 - 0.0205328\ \pi^2 x = 0$

$0.0051332\ x(x^2 - 4\ \pi^2) = 0.0051332\ x(x + 2\pi)(x - 2\pi) = 0$

.: stationary points are at $x = -2\pi$, 0 and 2π

Q3c and d



Q3e Area $= \int_{-2\pi}^{2\pi} (f(x) + 1 - g(x))dx \approx 0.837$ by CAS

Q3f $x \approx -3.400352 \approx -3.400$, or $x \approx 3.400$ by CAS

x-intercepts: $(-3.400, 0)$ and $(3.400, 0)$

Q3g $3.400352 \to \pi$, .: $m \approx \dfrac{\pi}{3.400352} \approx 0.923902 \approx 0.924$

Q3h $x \to \dfrac{x}{m}$, .: $p = m^4 \approx 0.729$ and $q = m^2 \approx 0.854$

Q4a $\int_0^{1.5} a\, x^2 (x - 1.5)^2 e^x\ dx = 1$,

$$a = \frac{1}{\int_0^{1.5} x^2 (x-1.5)^2 e^x\ dx} \approx \frac{1}{0.55773524} \approx 1.793 \text{ by CAS}$$

Q4b $\overline{X} = \int_0^{1.5} x\, f(x) dx \approx 0.82965 \approx 0.830$ by CAS

Q4c $\int_0^m \left(\dfrac{1}{0.55773524}\right) x^2 (x - 1.5)^2 e^x\ dx = 0.5$, $m \approx 0.843$ by CAS

Q4di Binomial: $n = 10$, $p = 0.137$

$\Pr(X < 3) = \Pr(X \le 2) \approx 0.8527664 \approx 0.853$ by CAS

Q4dii $\Pr(X < 3 \mid X \ge 1) = \dfrac{\Pr(1 \le X \le 2)}{1 - \Pr(X = 0)} \approx 0.8089999 \approx 0.809$

Q4e The mean of sample proportion $\approx p = 0.137$,

the standard deviation of sample proportion

$\approx \sqrt{\dfrac{p(1-p)}{n}} \approx 0.03438 \approx 0.034$

Q4f $\Pr(\hat{P} < 0.1 \mid \hat{P} < 0.2) = \dfrac{\Pr(\hat{P} < 0.1)}{\Pr(\hat{P} < 0.2)} \approx 0.1428 \approx 0.143$ by CAS

Q4g For sample size of 10, assuming the mean of sample proportion $\approx p = 0.137$, the standard deviation of sample proportion

$\approx \sqrt{\dfrac{p(1-p)}{n}} \approx 0.109$

$\Pr(\hat{P} < 0.1 \mid \hat{P} < 0.2) = \dfrac{\Pr(\hat{P} < 0.1)}{\Pr(\hat{P} < 0.2)} \approx 0.511$

For sample size of 200, the mean of sample proportion $\approx p = 0.137$,

the standard deviation of sample proportion $\approx \sqrt{\dfrac{p(1-p)}{n}} \approx 0.024$

$\Pr(\hat{P} < 0.1 \mid \hat{P} < 0.2) = \dfrac{\Pr(\hat{P} < 0.1)}{\Pr(\hat{P} < 0.2)} \approx 0.062$

$\Pr(\hat{P} < 0.1 \mid \hat{P} < 0.2) \approx 0.062$ is more reliable than

$\Pr(\hat{P} < 0.1 \mid \hat{P} < 0.2) \approx 0.511$ because the latter has a small sample size and normal approximation is not valid.

Q4h An approximate 95% confidence interval

$$\approx \left(0.137 - 1.96\sqrt{\frac{0.137(1-0.137)}{100}},\ 0.137 + 1.96\sqrt{\frac{0.137(1-0.137)}{100}}\right)$$

$\approx (0.070, 0.204)$

Please inform mathline@itute.com re conceptual and/or mathematical errors