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2017 NSW ESA Mathematics Exam Solutions

Section I

1	2	3	4	5	6	7	8	9	10
A	D	C	A	B	D	B	A	C	A

Q1 $y = -\frac{2}{3}x - \frac{4}{3}$ A

Q2 D

Q3 $\frac{d}{dx}e^{x^2} = \frac{d\left(e^{x^2}\right)}{d\left(x^2\right)} \times \frac{d\left(x^2\right)}{dx} = 2xe^{x^2}$ C

Q4 A

Q5 $\ln a = \ln b - \ln c = \ln\frac{b}{c}$, .: $a = \frac{b}{c}$ B

Q6 D

Q7 $\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta} = \frac{1}{\cos\theta\sin\theta} = \sec\theta\,\mathrm{cosec}\,\theta$ B

Q8 A

Q9 Line: $f'(x) = -2x + 4$, $y = f(x) = -x^2 + 4x + c$
When $x = 2$, $y = 12$, .: $c = 8$ C

Q10 The velocity changes from negative to positive once only when the graph crosses the horizontal axis. A

Section II

Q11a $\frac{2\left(\sqrt{5}+1\right)}{\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)} = \frac{2\left(\sqrt{5}+1\right)}{4} = \frac{\sqrt{5}+1}{2}$

Q11b $\int (2x+1)^4\,dx = \frac{(2x+1)^5}{5\times 2} + c = \frac{(2x+1)^5}{10} + c$

Q11c $\frac{d}{dx}\left(\frac{\sin x}{x}\right) = \frac{x\cos x - \sin x}{x^2}$

Q11d $\frac{d}{dx}\left(x^3 \ln x\right) = 3x^2 \ln x + x^3 \times \frac{1}{x} = x^2(3\ln x + 1)$

Q11ei Area of $\Delta OAB = \frac{1}{2} \times 6^2 \times \sin 30° = 9\text{ cm}^2$

Q11eii Area of shaded segment $= \frac{30}{360} \times \pi \times 6^2 - 9 = 3\pi - 9\text{ cm}^2$

Q11f $(x-2)^2 = 4a(y-1)$ passes through $(0, 4)$, .: $a = \frac{1}{3}$

.: $(x-2)^2 = 4\left(\frac{1}{3}\right)(y-1)$

Q11g $|3x-1| = 2$, $3x - 1 = \pm 2$, $3x = -1$ or 3, $x = -\frac{1}{3}$ or 1

Q11h $3 - x \ge 0$, $x \le 3$, .: the domain is $(-\infty, 3]$.

Q12a $y = x^2 + 4x - 7$, $\frac{dy}{dx} = 2x + 4 = 6$ at $(1, -2)$
Tangent: $y + 2 = 6(x-1)$, $y = 6x - 8$

Q12b $V = 2 \times \int_0^2 \pi y^2\,dx = 2\pi \int_0^2 \left(16 - 4x^2\right)dx$

$= 2\pi\left[16x - \frac{4x^3}{3}\right]_0^2 = \frac{128\pi}{3}$ cubic units

Q12c $a + 4d = 200$ and $2(2a + 3d) = 1200$, .: $a = 360$ and $d = -40$
.: the tenth term $= 360 + 9(-40) = 0$

Q12di $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \frac{|(1)(-4) + (-1)(0) + (-2)|}{\sqrt{1^2 + (-1)^2}} = 3\sqrt{2}$ units

Q12dii The length of $CD = \sqrt{3^2 + 3^2} = 3\sqrt{2}$ units
Area of the trapezium $= \frac{1}{2}\left(5\sqrt{2} + 3\sqrt{2}\right) \times 3\sqrt{2} = 24$ square units

Q12ei $\Pr(\text{even}) = \frac{2}{5}$

Q12eii $\Pr(\text{at least one even}) = 1 - \Pr(\text{none}) = 1 - \left(\frac{3}{5}\right)^3 = \frac{98}{125}$

Q12eiii $\Pr(\text{even odd odd}) = \left(\frac{2}{5}\right)\left(\frac{3}{5}\right)^2 = \frac{18}{125}$

Q12eiv $\Pr(\text{exactly one even}) = 3 \times \frac{18}{125} = \frac{54}{125}$

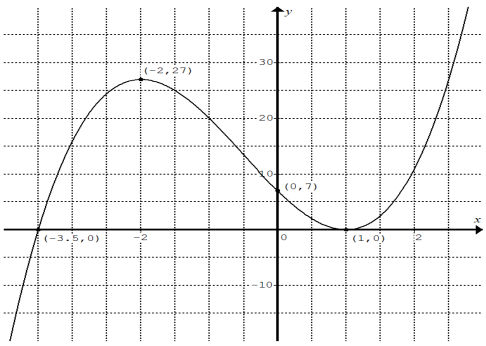
Q13a $(x-4)^2 + (x+4)^2 - 2(x-4)(x+4)\cos 60° = 13^2$ where $x - 4 > 0$, i.e. $x > 4$
$2x^2 + 32 - \left(x^2 - 16\right) = 169$, $x^2 = 121$, .: $x = 11$

Q13bi $y = 2x^3 + 3x^2 - 12x + 7$, $\frac{dy}{dx} = 6x^2 + 6x - 12 = 6(x+2)(x-1)$
To find the stationary points, let $\frac{dy}{dx} = 0$, .: $x = -2$ and $y = 27$ or $x = 1$ and $y = 0$.

x	< -2	-2	$-2 < x < 1$	1	> 1
dy/dx	positive	0	negative	0	positive
nature		max. t. p.		min. t. p.	

Stationary point $(-2, 27)$ is a maximum turning point, and stationary point $(1, 0)$ is a minimum turning point.

Q13bii $y = 2x^3 + 3x^2 - 12x + 7 = (x-1)^2(2x+7)$,
x-intercepts: at $x = -3.5$, $x = 1$, y-intercept: at $y = 7$



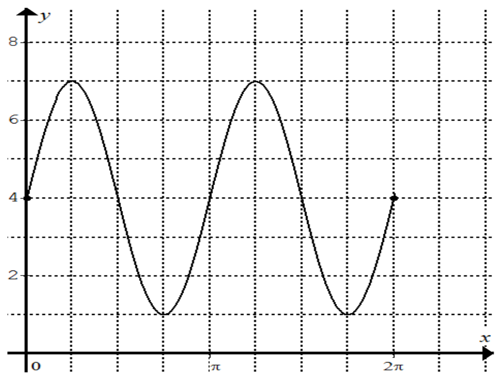
Q13biii $x < -2$ or $x > 1$

Q13c $t^{\frac{2}{3}} + t^{\frac{1}{3}} - 6 = 0$, $\left(t^{\frac{1}{3}} + 3\right)\left(t^{\frac{1}{3}} - 2\right) = 0$

.: $t^{\frac{1}{3}} + 3 = 0$ or $t^{\frac{1}{3}} - 2 = 0$, .: $t = -27$ or $t = 8$

Q13d $V = \int_0^{10} \frac{2t}{1+t^2}\,dt = \left[\ln(1+t^2)\right]_0^{10} = \ln(101)$

Q14a



Q14bi $\int_0^{\frac{\pi}{3}} \cos x\,dx = [\sin x]_0^{\pi/3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Q14bii $\int_0^{\frac{\pi}{3}} \cos x\,dx \approx \frac{\pi/3}{6}\left[\cos 0 + 4\cos\frac{\pi}{6} + \cos\frac{\pi}{3}\right] = \left(\frac{3+4\sqrt{3}}{36}\right)\pi$

Q14biii $\left(\frac{3+4\sqrt{3}}{36}\right)\pi \approx \frac{\sqrt{3}}{2}$, .: $\pi \approx \frac{18\sqrt{3}}{3+4\sqrt{3}}$

Q14ci $C = Ae^{kt}$, $\frac{dC}{dt} = kAe^{kt}$, $kC = kAe^{kt}$, .: $C = Ae^{kt}$ satisfies $\frac{dC}{dt} = kC$.

Q14cii Let $e^{k(5730)} = 0.5$, $5730k = \ln 0.5$, $k = \frac{\ln 0.5}{5730} \approx -0.00012$

Q14ciii $e^{kt} = 0.9$, $kt = \ln 0.9$, $t = \frac{5730 \times \ln 0.9}{\ln 0.5} \approx 8.7 \times 10^2$ years

Q14d $2 \times \int_0^1 \left[k(1-x^2) - 2k(x^2-1)\right]dx = 8$

$k \times \int_0^1 (1-x^2)\,dx = \frac{4}{3}$, .: $k\left[x - \frac{x^3}{3}\right]_0^1 = \frac{4}{3}$, .: $k = 2$

Q15a $\angle AED = 90° - \frac{x°}{2}$
For AB // ED, $\angle BAE + \angle AED = 180°$, sum of co-interior angles.
.: $3x + 90 - \frac{x}{2} = 180$, $x = 36$

Q15bi Amount ($) at the start of the first month after the deposit $= X$
Amount at the end of the first month after the withdrawal:
$M_1 = X\left(1 + \frac{0.042}{12}\right) - 2500 = 1.0035X - 2500$

Amount at the start of the second month after the deposit $= M_1 + X$
Amount at the end of the second month after the withdrawal:
$M_2 = (M_1 + X)1.0035 - 2500 = (1.0035X - 2500 + X)1.0035 - 2500$
$= X(1.0035^2 + 1.0035) - 2500(1.0035 + 1)$

Q15bii $M_3 = X(1.0035^3 + 1.0035^2 + 1.0035) - 2500(1.0035^2 + 1.0035 + 1)$
$M_{48} = X(1.0035^{48} + 1.0035^{47} + \ldots + 1.0035) - 2500(1.0035^{47} + \ldots + 1) = 80\,000$
$X = \frac{80\,000 + 2500(1.0035^{47} + \ldots + 1)}{(1.0035^{48} + 1.0035^{47} + \ldots + 1.0035)} \approx 4019.42$

Q15ci $v_1 = \int_0^t (6t + e^{-t})\,dt + 3 = \left[3t^2 - e^{-t}\right]_0^t + 3 = 3t^2 - e^{-t} + 4$

Q15cii $3t^2 - e^{-t} + 4 = 6t + 1 - e^{-t}$, $3t^2 - 6t + 3 = 0$, $3(t-1)^2 = 0$
.: $v_1 = v_2$ when $t = 1$

Q15ciii $x_1 = \int_0^t (3t^2 + 4 - e^{-t})\,dt = \left[t^3 + 4t + e^{-t}\right]_0^t = t^3 + 4t + e^{-t} - 1$

$x_2 = \int_0^t (6t + 1 - e^{-t})\,dt = \left[3t^2 + t + e^{-t}\right]_0^t = 3t^2 + t + e^{-t} - 1$, given $x = 0$ when $t = 0$ for both.
Let $t^3 + 4t + e^{-t} - 1 = 3t^2 + t + e^{-t} - 1$ for $t > 0$.
.: $t^3 - 3t^2 + 3t = 0$, $t(t^2 - 3t + 3) = 0$
The discriminant of $t^2 - 3t + 3 = 0$ is negative, .: no solution to the equation. The two particles do not meet for $t > 0$.

Q16ai

$L = \ell_{AE} + \ell_{EB} = \sqrt{x^2 + 5^2} + \sqrt{7^2 + (9-x)^2} = \sqrt{x^2 + 25} + \sqrt{49 + (9-x)^2}$

Q16aii

$\frac{dL}{dx} = \frac{x}{\sqrt{x^2+25}} - \frac{9-x}{\sqrt{49+(9-x)^2}} = 0$, $\frac{x}{\sqrt{x^2+25}} = \frac{9-x}{\sqrt{49+(9-x)^2}}$

$\therefore$ $\sin\alpha = \sin\beta$

Q16aiii The two triangles are similar, $\therefore$ $\frac{x}{5} = \frac{9-x}{7}$, $x = \frac{45}{12}$

Q16aiv If B is reflected in the line where the river runs, then AEB is a straight line, giving a minimum for L.

Q16b $\frac{a}{1-r} = 2$ for $-1 < r < 1$, $\therefore$ $-1 < -r < 1$, $0 < 1-r < 2$

$\therefore$ $0 < 2(1-r) < 4$, $\therefore$ $0 < a < 4$

Q16ci ΔBMD and ΔBCE are similar since $MD // CE$.

$\therefore$ $\frac{BM}{BD} = \frac{BC}{BE}$, $\therefore$ $\frac{BD}{BE} = \frac{BM}{BC} = \frac{1}{2}$ since M is the midpoint of BC

$\therefore$ D is the midpoint of BE

$\therefore$ $BD = DE = DA + AE$, and given $BD = DA + AC$

$\therefore$ $AC = AE$, $\therefore$ ΔACE is isosceles.

Q16cii $\angle BAF = \angle AEC$, corresponding angles are equal

$\angle FAC = \angle ACE$, alternate angles are equal

$\angle ACE = \angle AEC$, equal angles of an isosceles triangle

$\therefore$ $\angle BAF = \angle FAC$, $\therefore$ AF bisects $\angle BAC$

Please inform mathline@itute.com re conceptual and/or mathematical errors.