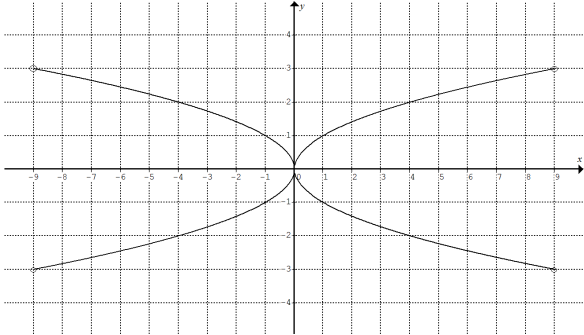


2017 Specialist Mathematics Trial Exam 1 Solutions

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Q1a



Q1b  $|y| = \sqrt{|x|}$ ,  $y^2 = |x|$ ,  $y^2 = \sqrt{x^2}$ ,

$2y \frac{dy}{dx} = \frac{x}{\sqrt{x^2}} = \frac{x}{|x|}$ ,  $\frac{dy}{dx} = \frac{x}{2y|x|} = \pm \frac{1}{2\sqrt{|x|}}$

Q1c  $(-9, 0) \cup (0, 9)$

Q2a  $\tilde{a} + \tilde{b} + \tilde{c} + \tilde{d} = \tilde{0}$ ,  $\therefore \tilde{d} = -(\tilde{a} + \tilde{b} + \tilde{c})$

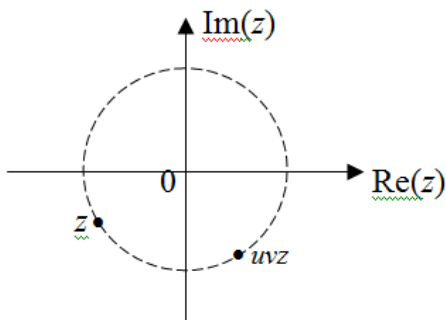
Q2b  $\vec{PQ} = \frac{1}{2}(\tilde{a} + \tilde{b}) = \frac{1}{2}\vec{AC}$ ,  $\vec{SR} = \frac{1}{2}(-\tilde{c} - \tilde{d}) = \frac{1}{2}\vec{AC}$

$\therefore \vec{PQ} = \vec{SR}$ ,  $\therefore PQRS$  is a parallelogram.

Q3a  $v = \frac{\bar{z}}{1-i}$ ,  $\bar{v} = \frac{z}{1+i}$ ,  $u\bar{v} = (1+i)z \times \frac{z}{1+i} = z^2$

Q3b  $uv = (1+i)z \times \frac{\bar{z}}{1-i} = \frac{(1+i)z\bar{z}}{1-i} = \frac{(1+i)^2 |z|^2}{2} = i$

Q3c  $uvz = iz$ ,  $uvz$  is the image of  $z$  after an anticlockwise rotation by  $90^\circ$  about  $O$ .



Q4a  $x = \sqrt{3} \sin 2t + \cos 2t$ ,  $\dot{x} = 2\sqrt{3} \cos 2t - 2 \sin 2t$

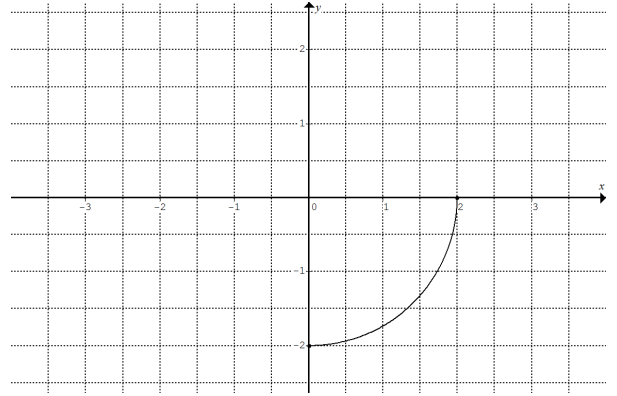
$\ddot{x} = -4\sqrt{3} \sin 2t - 4 \cos 2t$ , maximum speed occurs when  $\ddot{x} = 0$

$4\sqrt{3} \sin 2t = -4 \cos 2t$ ,  $\tan 2t = -\frac{1}{\sqrt{3}}$ ,  $2t = \frac{5\pi}{6}$ ,  $t = \frac{5\pi}{12}$

Q4b Max. speed =  $\left| \dot{x} \left( \frac{5\pi}{12} \right) \right| = \left| 2\sqrt{3} \cos \frac{5\pi}{6} - 2 \sin \frac{5\pi}{6} \right| = |-3-2| = 5$

Q5a  $x = 2 \sin 2t$ ,  $y = -2 \cos 2t$ ,  $\frac{x^2}{4} + \frac{y^2}{4} = 1$ ,  $x^2 + y^2 = 4$ ,

$0 \leq t \leq \frac{\pi}{4}$ ,  $0 \leq x \leq 2$ ,  $-2 \leq y \leq 0$

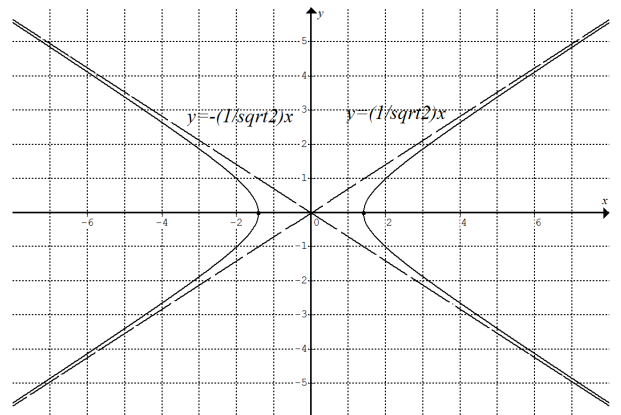


Q5b Arc length =  $\frac{1}{4} \times 2\pi \times 2 = \pi$  metres

Q6a  $\frac{dy}{dx} - \frac{x}{2y} = 0$ ,  $\int 2y dy = \int x dx$ ,  $y^2 = \frac{x^2}{2} + c$

$(2, 1)$  is on the curve,  $\therefore c = -1$  and  $y^2 = \frac{x^2}{2} - 1$  or  $\frac{x^2}{2} - y^2 = 1$

Q6b Hyperbola:  $x$ -intercepts are  $(-\sqrt{2}, 0)$  and  $(\sqrt{2}, 0)$ , asymptotes are  $y = \pm \frac{1}{\sqrt{2}}x$



Q7a  $\mu = \frac{3}{5} \times 32 + \frac{2}{5} \times 29 = 30.8$ ,  $E(\bar{X}) = \mu = 30.8$

Q7b  $\text{Var}(X) \approx \frac{3}{5} \times 8^2 + \frac{2}{5} \times 10^2 = 78.4$

$\sigma \approx \sqrt{78.4} \approx 8.854$ ,  $\text{sd}(\bar{X}) = \frac{\sigma}{\sqrt{10}} \approx 2.8$

Q8a All 6 roots lie on the unit circle centred at  $O$ , their arguments are separated by  $\frac{\pi}{3}$ .

Given  $z = -1$  is a root, then  $z = 1$  is also a root.

The others are:  $z = cis\left(\pm \frac{\pi}{3}\right) = \cos\left(\pm \frac{\pi}{3}\right) + i \sin\left(\pm \frac{\pi}{3}\right) = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

and  $z = cis\left(\pm \frac{2\pi}{3}\right) = \cos\left(\pm \frac{2\pi}{3}\right) + i \sin\left(\pm \frac{2\pi}{3}\right) = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

Q8b  $z - 2i = \pm 1, \frac{1}{2} \pm \frac{\sqrt{3}}{2}i, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

$z = \pm 1 + 2i, \frac{1}{2} + \left(2 \pm \frac{\sqrt{3}}{2}\right)i, -\frac{1}{2} + \left(2 \pm \frac{\sqrt{3}}{2}\right)i$

Q9a  $\tilde{s} = -2\tilde{p} + 3\tilde{q} - \tilde{r}$

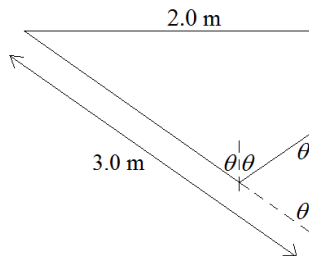
$= -2(2\tilde{i} + \tilde{j} + 2\tilde{k}) + 3(-\tilde{i} - 2\tilde{j} + 2\tilde{k}) - (2\tilde{i} - 2\tilde{j} - \tilde{k})$   
 $= -9\tilde{i} - 6\tilde{j} + 3\tilde{k}$

Q9b Let  $l(2\tilde{i} + \tilde{j} + 2\tilde{k}) + m(-\tilde{i} - 2\tilde{j} + 2\tilde{k}) + n(2\tilde{i} - 2\tilde{j} - \tilde{k}) = \tilde{0}$

$\therefore 2l - m + 2n = 0, l - 2m - 2n = 0$  and  $2l + 2m - n = 0$

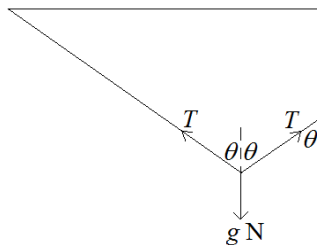
$\therefore l = m = n = 0, \therefore \tilde{p}, \tilde{q}$  and  $\tilde{r}$  are linearly independent vectors.

Q10a



Refer to the above diagram,  $\sin \theta = \frac{2}{3}$ .

Q10b



Refer to the above diagram,  $2T \cos \theta = g$

$2T \sqrt{1 - \sin^2 \theta} = g, 2T \sqrt{1 - \left(\frac{2}{3}\right)^2} = g, T = \frac{3\sqrt{5}}{10} g \text{ N}$

Please inform [mathline@itute.com](mailto:mathline@itute.com) re conceptual and/or mathematical errors