2017 VCAA Further Mathematics Exam 2 Solutions

SECTION A - Core
Data analysis

Q1ai Range $= 197 - 125 = 72$

Q1aii Percentage $= \frac{3}{12} \times 100\% = 25\%$

Q1bi Percentage $= \left(100 - \frac{100-68}{2}\right)\% = 84\%$

Q1bii $\left(100 - \frac{100-95}{2}\right)\%$ of $1000 = 975$

Q1c $\frac{x-165}{25} = -2.4$, $x = 105$

Q2a Place of capture

Q2b 20 mm

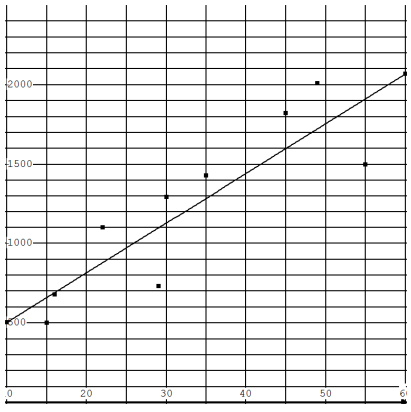
Q2c Forest minimum is 16; grassland Q_3 is 36

Q2d $Q_3 + 1.5 \times IQR = 32 + 1.5 \times (32 - 20) = 50$, $52 > 50$
∴ 52 is an outlier.

Q2e The back-to-back stem plot suggests that moths captured in grassland have a longer wingspan than those captured in forest as indicated by the median wingspans. $M_{forest} = 21$, $M_{grassland} = 30$

Q3a $egg\ density = -46.8 + 18.9 \times number\ of\ male\ moths$

Q3bi Two points on the line: $x = 10$, $y = 504$; $x = 60$, $y = 2069$



Q3bii The egg density increases by 31.3 eggs per square metre for each additional female moth caught in the trap.

Q3biii Predicted value $= 191 + 31.3 \times 55 \approx 1913$
Residual value = actual value − predicted value $= 1500 - 1913 \approx -413$

Q3biv $r^2 \approx 0.743$, ∴ 74.3%

Q4a 0.00854

Q4b $\log_{10}(area) = -14.4 + 0.00854 \times year$

Q4ci $\log_{10}(area) \approx 2.85$, $area \approx 708$ hectares

Q4cii The prediction was extrapolated too far from the known data.

Recursion and financial modelling

Q5a $V_1 = 75\,000 - 3375 = 71\,625$
$V_2 = 71\,625 - 3375 = 68\,250$

Q5bi \$3375

Q5bii $\frac{3375}{75\,000} = 0.045 = 4.5\%$

Q5c Annual depreciation percentage $= (1 - 0.943) \times 100\% = 5.7\%$

Q6a Amount paid $= \$200 \times 1.015 = \$203$

Q6b $A_{n+1} = 1.015 \times A_n$, $A_0 = \$428$

Q6c $A_4 = 1.015^4 \times \$428 = \$454.26$
Interest $\$454.26 - \$428 = \$26.26$

Q7a Monthly payment $= \$360\,000 \times \frac{0.052}{12} = \$1560$

Q7b First 4 years:
N = 48, I% = 3.8, PV = -360 000, PMT = -500, P/Y = 12, C/Y = 12,
FV = 444 872.9445
Next 2 years:
N = 24, I% = 3.8, PV = -444 872.9445, FV = 500 000, P/Y = 12, C/Y = 12,
PMT = -805.6505059
New monthly payment is \$805.65

SECTION B - Modules

Module 1: Matrices

Q1a $60+43+56=159$

Q1b $m_{12}=24$ indicates 24 rolls were sold in week 1.

Q1ci $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 35 & 24 & 60 \\ 28 & 32 & 43 \\ 32 & 30 & 56 \end{bmatrix}^{-1} \begin{bmatrix} 491.55 \\ 428.00 \\ 487.60 \end{bmatrix} = \begin{bmatrix} 4.65 \\ 4.20 \\ 3.80 \end{bmatrix}$

Cost of one sandwich = \$3.80

Q1cii $\begin{bmatrix} 0 & 1 & 1 \end{bmatrix}$

Q2a 40%

Q2b $300\times0.40+240\times0.60+210\times0.20=306$

Q2c $300+240+210=750$, same for each term

Q3a $0.4\times300=120$

Q3b $S_1=\begin{bmatrix} 300 \\ 200 \\ 200 \\ 300 \end{bmatrix}$, $S_2=TS_1=\begin{bmatrix} 250 \\ 250 \\ 300 \\ 200 \end{bmatrix}$

Q3c $S_3=TS_2=\begin{bmatrix} 260 \\ 240 \\ 295 \\ 205 \end{bmatrix}$

Number of students expected to choose (*I*) in term 3
$=240$
In this group, number of students chose (*S*) in term 2
$=0.3\times200=60$
$\therefore$ Percentage $=\frac{60}{240}\times100\%=25\%$

Q3d $S_4=TS_3=\begin{bmatrix} 261 \\ 239 \\ 294.5 \\ 205.5 \end{bmatrix}$

Maximum number of students at any time in 2018 is 250.

Module 2: Networks and decision mathematics

Q1a $15+105=120$ dollars

Q1b Quigley (*Q*) and Rosebush (*R*)

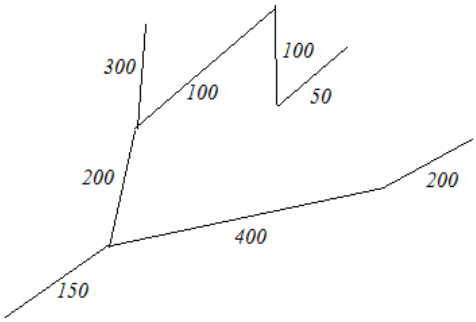
Q1c $6+7=11+2$

Q2a Colin will plan Tour 2

Q2b Agatha Tour 4, Bai Tour 1, Colin Tour 2, Diane Tour 3
Minimum total planning time $=10+7+8+18=43$ min

Q3ai A tree

Q3aii



Q4a *D* and *E*

Q4bi $A-E-I-L-N$

Q4bii $19-2-3-3-3-2=6$ days

Q4ci By crashing *I* (1 day) and *L* (1 day) or *J* (1 day) and *L* (1 day), the minimum number of days to complete the project is 17.

Q4cii Cost $=2\times\$1000=\$2000$

Module 3: Geometry and measurement

Q1ai Outside surface area $= 40\times19 = 760\,\text{cm}^2$

Q1aii Total outside surface area
$= 2(40\times19 + 32\times40 + 32\times19) = 5296\,\text{cm}^2$

Q1b Length $= 2\times40 = 80\,\text{cm}$

Q2a Thursday 12:00 midnight

Q2bi Distance $= \sqrt{186^2 + 50^2} \approx 193\,\text{m}$

Q2bii $\theta = \tan^{-1}\dfrac{50}{186} \approx 15°$, bearing $\approx 345°\,\text{T}$

Q2c Radius $= 6400\sin 43° \approx 4365\,\text{km}$

Q2d Shortest great circle distance $= \dfrac{43+38}{360}\times2\pi\times6400 \approx 9048\,\text{km}$

Q3a $65^2 = 50^2 + 50^2 - 2(50)(50)\cos\theta$

$\theta = \cos^{-1}\left(\dfrac{50^2+50^2-65^2}{2(50)(50)}\right) \approx 81°$

Q3b Perimeter $= \dfrac{81}{360}\times2\pi\times50 + 3\times65 \approx 266\,\text{m}$

Q3c Width of the segment $= 50 - 50\cos\left(\dfrac{81}{2}\right)° \approx 12\,\text{m}$

Length of path AB $\approx 50 + 50 + 65 - 12 = 153\,\text{m}$

Module 4: Graphs and relations

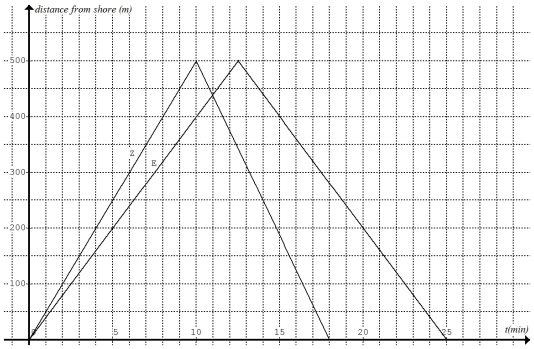
Q1a 2 m

Q1b Between 8:00 am and 6:00 pm

Q2a $500 + 100 = 600\,\text{m}$

Q2b $k = \dfrac{500}{12.5} = 40$

Q2c



Q2d Let $40t = -62.5t + 1125$, $t = 10.98$ minutes

Q3a $y \geq \dfrac{8}{4} = 2$, .: 2 senior lifeguards

Q3b At least 12 lifeguards (junior and senior) are required.

Q3c $(8, 4)$ will result in minimum total meal allowance.
$C_{\min} = 15\times8 + 25\times4 = \$220$

Q3d Junior lifeguards 9 Senior lifeguard 3

$(8, 4)$ will result in minimum total meal allowance.
$C_{\min} = 15\times8 + 25\times4 = \$220$

Please inform mathline@itute.com re conceptual and/or mathematical errors