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2017 VCAA Mathematical Methods Exam 1 Solutions

Q1a $f'(x)=\dfrac{(x+2)(1)-x(1)}{(x+2)^2}=\dfrac{2}{(x+2)^2}$ for $x>-2$

Q1b $g'(x)=3\left(2-x^3\right)^2\left(-3x^2\right)$, $g'(1)=3(1)^2(-3)=-9$

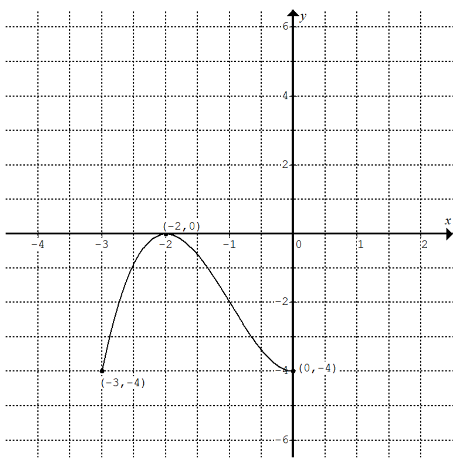
Q2a $\dfrac{dy}{dx}=\dfrac{d}{dx}\left(x\log_e(3x)\right)=(1)\log_e(3x)+x\times\dfrac{1}{x}=\log_e(3x)+1$

Q2b $\int_1^2\left(\log_e(3x)+1\right)dx=\left[x\log_e(3x)\right]_1^2=2\log_e 6-\log_e 3$

$=\log_e 6^2-\log_e 3=\log_e\dfrac{36}{3}=\log_e 12$

Q3a $(x+2)^2(x-1)=\left(x^2+4x+4\right)(x-1)=x^3+3x^2-4$

Q3b Let $f'(x)=3x^2+6x=3x(x+2)=0$ for stationary points.
.: $x=-2$ or 0, .: $y=0$ or -4 respectively.



Q4 $\mathrm{sd}\left(\hat{P}\right)=\sqrt{\dfrac{p(1-p)}{n}}$, $\sqrt{\dfrac{\frac{1}{4}\left(1-\frac{1}{4}\right)}{n}}\le\dfrac{1}{100}$, $\dfrac{\frac{1}{4}\left(1-\frac{1}{4}\right)}{n}\le\dfrac{1}{10000}$,

.: $\dfrac{3}{16}\times 10000\le n$, $n\ge 1875$, .: the smallest value of n is 1875.

Q5a $\Pr(\text{not log on})=\Pr(FFF)=\left(\dfrac{3}{5}\right)^3=\dfrac{27}{125}$

Q5b $\Pr(\text{log on})=1-\Pr(\text{not log on})=1-\left(\dfrac{3}{5}\right)^3=\dfrac{98}{125}$

Q5c $\Pr(FS)+\Pr(FFS)=\dfrac{3}{5}\times\dfrac{2}{5}+\dfrac{3}{5}\times\dfrac{3}{5}\times\dfrac{2}{5}=\dfrac{48}{125}$

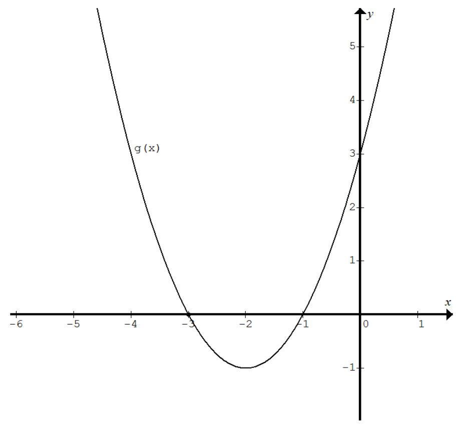
Q6a $\tan\theta=1$ or $\pm\sqrt{3}$

Q6b Given $0\le\theta\le\pi$ and $\tan\theta=1$, $\theta=\dfrac{\pi}{4}$,

also $\tan\theta=\pm\sqrt{3}$, $\theta=\dfrac{\pi}{3}$ or $\dfrac{2\pi}{3}$.

Q7a $f(0)=1$, the range of f is $[1,\infty)$.

Q7bi



For the range of $g\subseteq[0,\infty)$, the largest c is -3.

Q7bii For the value of c found in part bi, $0\le g(x)\le\infty$, .: the range of $f(g(x))$ is $[1,\infty)$.

Q7c $3\le h(x)\le\infty$, $f(3)=2$, .: the range of $f(h(x))$ is $[2,\infty)$.

Q8a $\Pr(B\mid A)=\dfrac{\Pr(A\cap B)}{\Pr(A)}=\dfrac{p}{\Pr(A)}=\dfrac{1}{4}$, .: $\Pr(A)=4p$ and $p>0$

Q8b $\Pr(A\mid B)=\dfrac{\Pr(A\cap B)}{\Pr(B)}=\dfrac{p}{\Pr(B)}=\dfrac{1}{5}$, .: $\Pr(B)=5p$

$\Pr(A'\cap B')=\Pr\left((A\cup B)'\right)=1-\Pr(A\cup B)$

$=1-\left(\Pr(A)+\Pr(B)-\Pr(A\cap B)\right)=1-(4p+5p-p)=1-8p$

Q8c $\Pr(A\cup B)\le\dfrac{1}{5}$, $8p\le\dfrac{1}{5}$, .: $0<p\le\dfrac{1}{40}$

Q9a

$\text{Area}=\int_0^1\sqrt{x}\,(1-x)dx=\int_0^1\left(x^{\frac{1}{2}}-x^{\frac{3}{2}}\right)dx=\left[\dfrac{2x^{\frac{3}{2}}}{3}-\dfrac{2x^{\frac{5}{2}}}{5}\right]_0^1=\dfrac{2}{3}-\dfrac{2}{5}=\dfrac{4}{15}$

Q9b $f'(x)=\sqrt{x}(-1)+\dfrac{1-x}{2\sqrt{x}}=\dfrac{1-3x}{2\sqrt{x}}$

Q9c $m_{AC} = \tan 45° = 1$, .: $m_{BC} = -1$

Let $\frac{1-3x}{2\sqrt{x}} = -1$, $3x - 2\sqrt{x} = 1$, .: $x = 1$ by inspection.

.: BC is tangent to the curve at $(1, 0)$ and cuts the y-axis at $(0, 1)$.

Equation of BC is $y = 1 - x$.

Q9d Let $\frac{1-3x}{2\sqrt{x}} = 1$, $3x + 2\sqrt{x} - 1 = 0$, .: $(3\sqrt{x} - 1)(\sqrt{x} + 1) = 0$

Since $\sqrt{x} + 1 > 0$, .: $3\sqrt{x} - 1 = 0$, .: $x = \frac{1}{9}$ and $y = \frac{1}{3}\left(1 - \frac{1}{9}\right) = \frac{8}{27}$

Let $A(a, 0)$, and knowing $m_{AC} = 1$, .: $\frac{\frac{8}{27} - 0}{\frac{1}{9} - a} = 1$, .: $a = -\frac{5}{27}$

ΔABC is an isosceles triangle, .: $x_C = \frac{1 - \frac{5}{27}}{2} = \frac{11}{27}$

.: $y_C = 1 - x_C = \frac{16}{27}$, .: $C\left(\frac{11}{27}, \frac{16}{27}\right)$

Please inform mathline@itute.com re conceptual and/or mathematical errors