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2017 VCAA Mathematical Methods Exam 2 Solutions

Note: Use CAS to save time

SECTION A – Multiple-choice questions

1	2	3	4	5	6	7	8	9	10
C	D	C	E	A	C	B	A	A	D

11	12	13	14	15	16	17	18	19	20
D	C	E	D	B	C	D	D	D	B

Q1 Period $=\frac{2\pi}{2}=\pi$, range is $[-1-5,-1+5]=[-6,4]$ C

Q2 D

Q3 $\frac{\binom{5}{1}\binom{3}{1}}{\binom{8}{2}}=\frac{15}{28}$ C

Q4 $f(g(3))=f(2)=5$ E

Q5 $\hat{p}+2\times \text{sd}=0.121$, $\hat{p}+\frac{0.121-0.039}{2}=0.121$, $\hat{p}=0.080$ A

Q6 C

Q7 $(p-1)x^2+4x+(p-5)=0$, $\Delta=16-4(p-1)(p-5)<0$

$\therefore\ p^2-6p+1>0$ B

Q8 $a^{b-4x}=y-2$, $b-4x=\log_a(y-2)$, $x=\frac{1}{4}(b-\log_a(y-2))$ A

Q9 Av. rate $=\frac{f(a)-f(1)}{a-1}=\frac{a^2-2a+1}{a-1}=8$, $a>1$, $\therefore\ a=9$ A

Q10 $x=\frac{x'}{2}$, $y=3y'$

$y=3\sin 2\left(x+\frac{\pi}{4}\right)=3\cos 2x\ \rightarrow 3y'=3\cos x'$, i.e. $y=\cos x$ D

Q11 $f'(x)=3x^2+2ax+b$

$f'(-1)=3-2a+b=0$ and $f'(3)=27+6a+b=0$

$\therefore\ a=-3$ and $b=-9$ D

Q12 $\sin 2x=\frac{\sqrt{3}}{2}$, $x=-\frac{5\pi}{6},-\frac{2\pi}{3},\frac{\pi}{6},\frac{\pi}{3},\frac{7\pi}{6},\ldots$

$\frac{-5\pi}{6}+\frac{-2\pi}{3}+\frac{\pi}{6}+\frac{\pi}{3}=-\pi$, $\therefore\ \frac{\pi}{3}\le x<\frac{7\pi}{6}$ C

Q13 $(h(x))^2=\frac{1}{(x-1)^2}$, $h(x^2)=\frac{1}{x^2-1}$, $\therefore\ (h(x))^2\ne h(x^2)$ E

Q14 $\mu=1-4p$, $\text{Var}(X)=\text{E}(X^2)-\mu^2=6p-16p^2$ D

Q15 $\text{A}=uv=u(-u^3+8)=-u^4+8u$

Let $\frac{d\text{A}}{du}=-4u^3+8=0$, $\therefore\ u=\sqrt[3]{2}$, $\text{A}_{\max}=6\sqrt[3]{2}$ B

Q16 $\hat{P}=\frac{X}{n}=0$, $\therefore\ X=0$

$\Pr(\hat{P}=0)=\Pr(X=0)=(1-p)^5=\frac{1}{243}$ $\therefore\ p=\frac{2}{3}$

$\hat{P}=\frac{X}{n}=\frac{X}{5}>0.6$, $\therefore\ X>3$

$\Pr(\hat{P}>0.6)=\Pr(X>3)=\Pr(X=4)+\Pr(X=5)=0.4609$ C

Q17 $b+c=0$ D

Q18 $\mu=\sigma$, $np=\sqrt{np(1-p)}$, $(np)^2=np(1-p)$, $np=1-p$

$\therefore\ p=\frac{1}{n+1}\le 0.01$, $n+1\ge 100$, $n\ge 99$ D

Q19 $\int_k^{k+1}(\cos x+1)dx=1$, $[\sin x+x]_k^{k+1}=1$, $\sin(k+1)-\sin k=0$

For $0<k<2$, $\sin(k+1)-\sin(\pi-k)=0$

$\therefore\ k+1=\pi-k$, $k=\frac{\pi-1}{2}$ D

Q20 Let $\sqrt{3}\sin x=\cos x$, $\tan x=\frac{1}{\sqrt{3}}$, $x=\frac{\pi}{6}$, $\therefore\ B\left(\frac{\pi}{6},\frac{\sqrt{3}}{2}\right)$

Area of $\Delta OAB=\frac{1}{2}\times\frac{\pi}{2}\times\frac{\sqrt{3}}{2}=\frac{\sqrt{3}\pi}{8}$

Area of the shaded region $=\int_0^{\frac{\pi}{6}}\sqrt{3}\sin x\,dx+\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\cos x\,dx=\sqrt{3}-1$ B

SECTION B

Q1a $f(x)=x^3-5x$

Let $f'(x)=3x^2-5=0$, $x=\pm\sqrt{\frac{5}{3}}$, $y=\mp\frac{10}{3}\sqrt{\frac{5}{3}}$

The turning points are $\left(-\sqrt{\frac{5}{3}},\ \frac{10}{3}\sqrt{\frac{5}{3}}\right)$ and $\left(\sqrt{\frac{5}{3}},\ -\frac{10}{3}\sqrt{\frac{5}{3}}\right)$

Q1bi The line *AB* passes through the origin.

$m_{AB}=\frac{f(1)-f(-1)}{1-(-1)}=-4$.: the equation of the line is $y=-4x$

Q1bii $\overline{AB}=2\sqrt{1^2+4^2}=2\sqrt{17}$

Q1ci $\overline{CD}=2\sqrt{1^2+(k-1)^2}=2\sqrt{k^2-2k+2}$

Q1cii Let $2\sqrt{k^2-2k+2}=k+1$, .: $k=1$ or $\frac{7}{3}$

Q1di $a^3-ak=a$, $a>0$, $a^2-k=1$, $a=\sqrt{k+1}$

Q1dii Area of the shaded region $=\int_0^{\sqrt{k+1}}\left(x-\left(x^3-kx\right)\right)dx=\frac{(k+1)^2}{4}$

Q2a $h_{\min}=65-55=10$, $h_{\max}=65+55=120$

Q2b Time in the capsule = one period $=\frac{2\pi}{\frac{\pi}{15}}=30$ minutes

Q2c Rate of change of $h=h'(t)=\frac{11\pi}{3}\sin\left(\frac{\pi t}{15}\right)$

Let $h''(t)=\frac{11\pi^2}{45}\cos\left(\frac{\pi t}{15}\right)=0$, $t=\frac{15}{2}$

Q2d $\tan\theta=\frac{65}{500}$, $\theta=7.41°$

Q2e $\frac{dy}{dx}=-\frac{x}{\sqrt{3025-x^2}}$

Q2f $m_{P_2B}=\frac{-v}{500-u}=-\frac{\sqrt{3025-u^2}+65}{500-u}$

Let $-\frac{\sqrt{3025-u^2}+65}{500-u}=-\frac{u}{\sqrt{3025-u^2}}$

.: $u=13.00$ and $v=\sqrt{3025-u^2}+65=118.44$

.: $P_2(13.00, 118.44)$

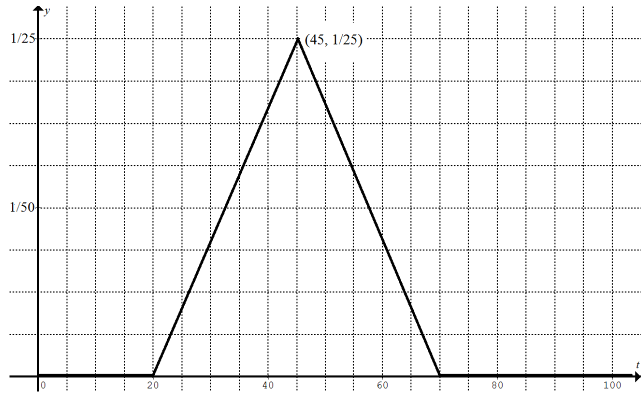
Q2g $\tan\alpha=\frac{118.44}{500-13.00}$, $\alpha\approx 13.67°$

Q2h From P_1 to P_2, point *P* has rotated

$(180-13.67)°-(90-7.41)°=83.74°$

Length of time $\approx\frac{83.74}{360}\times 30\approx 7$ min

Q3a



Q3b $f(25)=\frac{5}{625}$, $f(55)=\frac{15}{625}$

$\Pr(25\le T\le 55)=1-\frac{1}{2}(25-20)f(25)-\frac{1}{2}(70-55)f(15)$

$=1-0.02-0.18=0.80$

Q3c $\Pr(T\le 25\,|\,T\le 55)=\frac{\Pr(T\le 25)}{\Pr(T\le 55)}=\frac{0.02}{0.80+0.02}=\frac{1}{41}$

Q3d $f(a)=\frac{1}{625}(a-20)$, $\Pr(T\ge a)=0.7$, $\Pr(T\le a)=0.3$

.: $\frac{1}{2}(a-20)\times\frac{1}{625}(a-20)=0.3$, $a\approx 39.3649$

Q3ei Binomial: $n=7$, $p=\frac{8}{25}$, $\Pr(X>3)=\Pr(X\ge 4)\approx 0.1534$

Q3eii $\Pr(X\ge 2\,|\,X\ge 1)=\frac{\Pr(X\ge 2)}{\Pr(X\ge 1)}\approx 0.7626$

Q3f $q=\Pr(X=2)+\Pr(X=3)=\binom{7}{2}p^2(1-p)^5+\binom{7}{3}p^3(1-p)^4$

.: $q=14p^7-35p^6+70p^4-70p^3+21p^2$

Q3gi $p\approx 0.3539$ and $q\approx 0.5665$

Q3gii $\Pr(T>d)\approx 0.3539$, $\frac{1}{2}(70-d)\times\frac{1}{625}(70-d)\approx 0.3539$

$d\approx 49$

Q4a $c=-1$ and $d=-2$

Q4b Domain for f^{-1} is $(-2,\infty)$.

Let $x=2^{y+1}-2$, $y=\log_2(x+2)-1$, $f^{-1}(x)=\log_2(x+2)-1$

Q4c The two graphs intersect at $y=x=2^{x+1}-2$, i.e. $x=-1$

Area $=2\times\int_{-1}^{0}\left(x-\left(2^{x+1}-2\right)\right)dx=2\times\int_{-1}^{0}\left(x+2-2^{x+1}\right)dx$

$=2\times\int_{-1}^{0}\left(x+2-e^{(\log_e 2)(x+1)}\right)dx=3-\frac{2}{\log_e 2}$

Q4d $f'(0)=2\log_e 2$, $\left(f^{-1}\right)'(0)=\frac{1}{f'(0)}=\frac{1}{2\log_e 2}$

Q4e Let $2e^{kx}-2=2^{x+1}-2$, $e^{kx}=2^x$, .: $k=\log_e 2$

Q4f Let $x=2e^{ky}-2$, $y=\frac{1}{k}\log_e\left(\frac{x+2}{2}\right)$

.: $g_k^{-1}(x)=\frac{1}{k}\log_e\left(\frac{x+2}{2}\right)$

Q4gi $g_1=2e^x-2 \rightarrow g_1=2e^{kx}-2$

Dilation by a factor of $\frac{1}{k}$ in the direction parallel to the x-axis

Q4gii

Dilation by a factor of $\frac{1}{k}$ in the direction parallel to the y-axis

Q4h Let $m_1=\frac{dg}{dx}=2ke^{kx}=2k$ at $x=0$, .: $m_2=\frac{1}{2k}$ where $k>0$

$\tan\theta=\pm\frac{m_1-m_2}{1+m_1m_2}$, $\tan 30°=\pm\frac{2k-\frac{1}{2k}}{2}$, $\frac{1}{\sqrt{3}}=\pm\frac{2k-\frac{1}{2k}}{2}$

.: $k=\frac{\sqrt{3}}{6}$ or $\frac{\sqrt{3}}{2}$

Q4i i Same gradient at $x=0$, .: $2p=\frac{1}{2p}=1$, .: $p=\frac{1}{2}$

Q4i ii As $k\rightarrow\infty$, the graph of g_k approaches the lines $x=0$ and $y=-2$, and the graph of g_k^{-1} approaches the lines $y=0$ and $x=-2$.

.: $b=4$, the area of the square region enclosed by $x=-2$, $x=0$, $y=-2$ and $y=0$.

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