2018
Further
Mathematics

Year 12
Problem Solving Task

Time allowed: 2 hours plus

Theme: Speed and distance travelled in a car trip

Required skills/knowledge: Linear and piecewise relations, inequalities, simultaneous linear equations, graphs and modelling.

The task: Based on the given information, equations and inequalities are formulated to model practical situations and solve related problems.

Students also investigate the effects of changing some conditions/parameters on the practical situations and solutions to the problems.

Start of task:

Part I

Trevor travelled by car from his home (the starting point) and recorded the distance travelled along the route in km every 0.10 hour (i.e. 6 min). He followed the same route to go home. A line segment graph of the distance from home against time for part of the trip is shown below.

(1) *Determine the maximum distance travelled in the first 2 hours.*

(2) *For how long (in min) did he exceed the 100 km per hour speed limit?*

(3) *What was his average speed in the first hour of the trip?*

(4)   *What was his average speed in the second hour of the trip?*

(5)   *Determine the shortest time required (from $t = 2$) to return home in order not to exceed the speed limit.*

(6)   *Determine the piecewise relation for Trevor's car trip from home back to home, 6 line segments altogether including the line segment for the last part of the trip (from $t = 2$) which does not appear in the graph above. The first one is shown in the piecewise relation below.*

$$
d = \begin{cases}
90t & 0 \le t \le 0.5 \\
\underline{\qquad} & \underline{\qquad} \\
\underline{\qquad} & \underline{\qquad} \\
\underline{\qquad} & \underline{\qquad} \\
\underline{\qquad} & \underline{\qquad} \\
\underline{\qquad} & \underline{\qquad}
\end{cases}
$$

Trevor plans to go on a second trip along the same road covering 200 km from home back to home in 2.5 hours of continuous driving without exceeding the speed limit. This time the trip has 3 parts (line segments).

(7)   *Sketch a graph of 3 line segments below, satisfying the requirements specified above.*
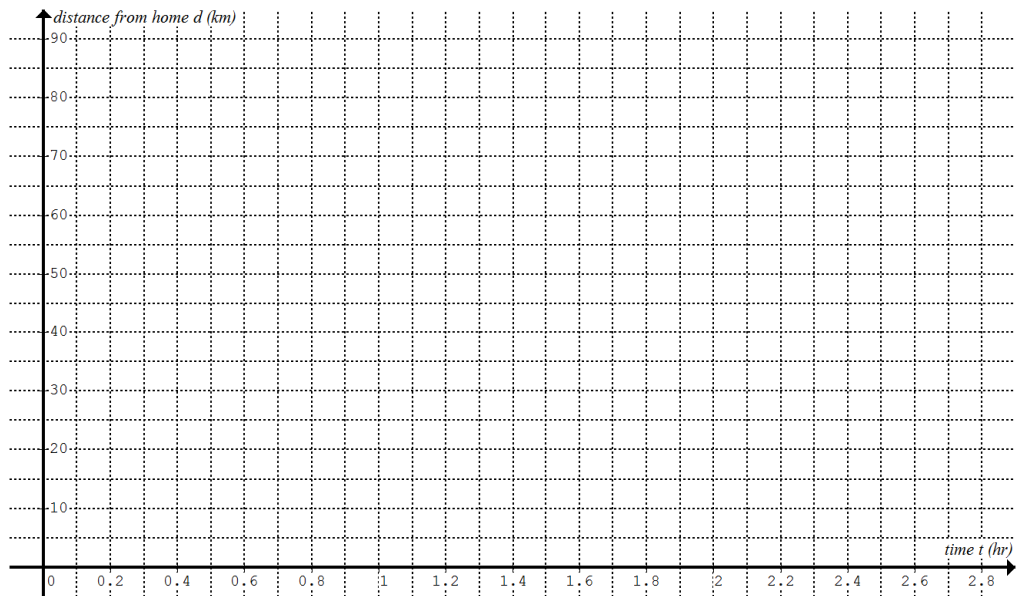
(8)   *Show that each part of the trip does not exceed the speed limit.*

(9)   Assumptions are made when line segments are used in the distance-time graphs above. State the assumptions and discuss whether the assumptions are realistic or not.

Part II

A challenge

Trevor's housemate challenges him to drive from home as far as possible along the same road and return home in 2 hours under the following constraints.
- A. The 100 km per hour speed limit is not to be exceeded.
- B. He must travel at most v km per hour for at least t hours.

(10) Determine the furthest distance from home that can be achieved by Trevor if $v = 80$ and $t = 0.5$.

(11) Help Trevor to plan the trip to meet the housemate's challenge.
Specify the trip plan as a piecewise relation.
Sketch the graph of the piecewise relation in the grid provided below.
Specify the other possible plans, if any, as piecewise relations. Notes: Graphs are not required.
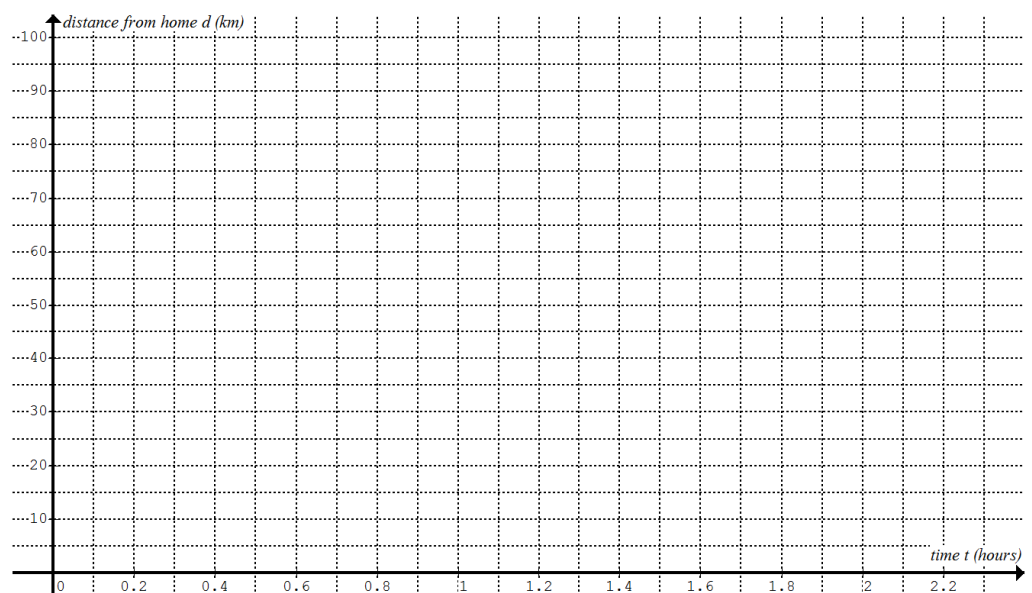
(12)   Systematically investigate the effects on the furthest distance from home if $t = 0.5$ and v varies between 70 and 90 inclusively, i.e. $70 \le v \le 90$. Describe the effects.

(13)   Systematically investigate the effects on the furthest distance from home if $v = 80$ and t varies between 0.3 and 0.8 inclusively, i.e. $0.3 \le t \le 0.8$. Describe the effects.

End of Problem Solving Task