2018 NSW ESA Mathematics Exam Solutions

Section I

1	2	3	4	5	6	7	8	9	10
B	C	A	D	D	C	C	D	B	D

Q1 B

Q2 C

Q3 A

Q4 By sketching a rough graph of the line $3x-4y+3=0$ and mark down the centre of the circle, it is obvious the radius of the circle is greater than 2. D

Q5 D

Q6 $\dfrac{4}{\binom{8}{2}}=\dfrac{1}{7}$ C

Q7 $\int_0^3 f(x)dx=10+3=13$, $\int_{-1}^3 f(x)dx=-2+13=11$ C

Q8 $x^2=4ay$, $12^2=4a4$, $a=9$ D

Q9 Turning point of $f'(x)$ B

Q10 Sketch the 4 graphs. D

Section II

Q11a $\dfrac{3(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})}=\dfrac{3(3-\sqrt{2})}{7}$

Q11b $1-3x>10$, $-9>3x$, $x<-3$

Q11c $\dfrac{(2x)^3-(3y)^3}{(2x-3y)}=\dfrac{(2x-3y)(4x^2+6xy+9y^2)}{(2x-3y)}=4x^2+6xy+9y^2$

Q11di $a+2d=8$, $a+19d=59$, .: $17d=51$, $d=3$

Q11dii $t_{50}-t_{20}=30d$, $t_{50}-59=30\times 3$, $t_{50}=149$

Q11e $\int_0^3 e^{5x}dx=\left[\dfrac{e^{5x}}{5}\right]_0^3=\dfrac{1}{5}(e^{15}-1)$

Q11f $\dfrac{d}{dx}x^2\tan x=2x\tan x+x^2\sec^2 x$

Q11g $\dfrac{d}{dx}\dfrac{e^x}{x+1}=\dfrac{(x+1)e^x-e^x}{(x+1)^2}=\dfrac{xe^x}{(x+1)^2}$

Q12ai $\angle ABC=50°+(180-120)°=110°$

Q12aii $AC=\sqrt{320^2+190^2-2(320)(190)\cos 110°}\approx 420$ km

Q12b $x=\dfrac{\pi}{6}$, $y=\cos\dfrac{\pi}{3}=\dfrac{1}{2}$, $m=\dfrac{dy}{dx}=-2\sin 2x=-\sqrt{3}$

Tangent: $\left(y-\dfrac{1}{2}\right)=-\sqrt{3}\left(x-\dfrac{\pi}{6}\right)$, $y=-\sqrt{3}\left(x-\dfrac{\pi}{6}\right)+\dfrac{1}{2}$

Q12ci $AB=AD$, $BE=DF$, $\angle B=\angle D=90°$, SAS, congruent

Q12cii Area $=14^2-14\times 10=56$ cm^2

Q12di $t=0$, $v=\dfrac{dx}{dt}=t^2-4t+3=3$

Q12dii $v=\dfrac{dx}{dt}=t^2-4t+3=0$, $t=1, 3$

Q12diii $a=\dfrac{dv}{dt}=2t-4=0$, $t=2$, $x=\dfrac{2^3}{3}-2\times 2^2+3\times 2=\dfrac{2}{3}$

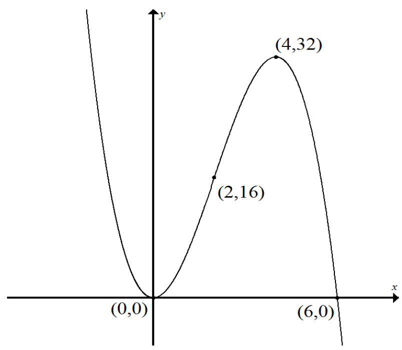
Q13ai $\dfrac{dy}{dx}=12x-3x^2=0$, $x=0$, 4. Stationary points $(0,0)$, $(4,32)$

x	<0	0	$0<x<4$	4	>4
dy/dx	negative	0	positive	0	negative
Nature		min t p		max t p	

Q13aii $y=16$ and $\dfrac{d^2y}{dx^2}=12-6x=0$ when $x=2$,

x	2^-	2	2^+
dy/dx	positive	12	positive
Nature		inflection p	

Q13aiii



Q13bi Common $\angle B$, isosceles triangles, .: $LBDC=\angle BCA$, .: $\angle BCD=\angle A$, AAA, .: the two isosceles triangles are similar.

Q13bii Similar triangles, .: $\dfrac{BD}{2}=\dfrac{2}{3}$, $BD=\dfrac{4}{3}$

$AD=AB-BD=3-\dfrac{4}{3}=\dfrac{5}{3}$

Q13ci 1960 was 50 years after 1910.
$184=92e^{50k}$, $50k=\log_e 2$, $k\approx 0.0139$

Q13cii 2020 will be 110 years after 1910.
$P(110)=92e^{0.0139\times 110}\approx 424$ millions

Q14ai Area $=\frac{1}{2}\times 3\times 6\times \sin 60° = \frac{9\sqrt{3}}{2}$

Q14aii $\frac{1}{2}\times 3\times x\times \sin 30° + \frac{1}{2}\times 6\times x\times \sin 30° = \frac{9\sqrt{3}}{2}$

$\frac{3x}{2}+\frac{6x}{2}=9\sqrt{3}$, $x = 2\sqrt{3}$

Q14b $x^2 = \sqrt{y-1}$

Volume $=\int_1^{10}\pi x^2 dy = \int_1^{10}\pi\sqrt{y-1}\,dy = \pi\left[\frac{2(y-1)^{\frac{3}{2}}}{3}\right]_1^{10} = 18\pi$

Q14c $f'(x) = 3x^2 + 2kx + 3 \neq 0$ when $\Delta = 4k^2 - 36 < 0$

.: $k^2 < 9$, .: $-3 < k < 3$

Q14di 1st day: $2^1 + 1 = 3$, 2nd day: $2^2 + 2 = 6$, 3rd day: $2^3 + 3 = 11$

Q14dii Total $=\left(2^1 + 2^2 + 2^3 + ... + 2^{20}\right) + (1+2+3+...+20)$

$=\frac{2\left(2^{20}-1\right)}{2-1} + \frac{20}{2}(1+20) = 2097360$

Q14ei Pr(at least one is faulty) $= 1 -$ Pr(none) $= 1 - 0.9\times 0.95 = 0.145$

Q14eii Pr(neither is faulty) $= 0.5\times 0.9^2 + 0.5\times 0.95^2 = 0.85625$

Q15ai $L(0) = 12 + 2\cos 0 = 14$

Q15aii Shortest $= 12 - 2 = 10$

Q15aiii $12 + 2\cos\frac{2\pi t}{366} = 11$, $\cos\frac{2\pi t}{366} = -\frac{1}{2}$

$\frac{2\pi t}{366} = \frac{2\pi}{3}, \frac{4\pi}{3}$, .: $t = 122, 244$

Q15b $\int_0^k \frac{1}{x+3}dx = \int_k^{45}\frac{1}{x+3}dx$, $\left[\log_e(x+3)\right]_0^k = \left[\log_e(x+3)\right]_k^{45}$

$\log_e\frac{k+3}{3} = \log_e\frac{48}{k+3}$, $\frac{k+3}{3} = \frac{48}{k+3}$, $(k+3)^2 = 144$, $k = 9$

Q15ci Area $=\int_0^3\left(2x - \left(x^3 - 7x\right)\right)dx = \int_0^3\left(9x - x^3\right)dx$

$=\left[\frac{9x^2}{2} - \frac{x^4}{4}\right]_0^3 = \frac{81}{4}$

Q15cii

Area $=\int_0^3\left(9x - x^3\right)dx = \frac{3-0}{6}\left[0 + 4\left(9\times\frac{3}{2} - \left(\frac{3}{2}\right)^3\right) + (27-27)\right]$

$=2\left(\frac{27}{2} - \frac{27}{8}\right) = \frac{81}{4}$

Q15ciii Let $\frac{dy}{dx} = 3x^2 - 7 = 2$, $x = \sqrt{3}$, $y = \left(\sqrt{3}\right)^3 - 7\sqrt{3} = -4\sqrt{3}$

.: $P\left(\sqrt{3}, -4\sqrt{3}\right)$

Q15civ $d = \frac{2\sqrt{3} + (-1)\left(-4\sqrt{3}\right)}{\sqrt{2^2 + (-1)^2}} = \frac{6\sqrt{3}}{\sqrt{5}}$, $OA = \sqrt{3^2 + 6^2} = \sqrt{45} = 3\sqrt{5}$

Area $=\frac{1}{2}\times 3\sqrt{5}\times\frac{6\sqrt{3}}{\sqrt{5}} = 9\sqrt{3}$

Q16ai $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi x^2\sqrt{10^2 - x^2} = \frac{1}{3}\pi x^2\sqrt{100 - x^2}$

Q16aii $\frac{dV}{dx} = \frac{\pi}{3}\left(2x\sqrt{100-x^2} - \frac{2xx^2}{2\sqrt{100-x^2}}\right)$

$=\frac{\pi x}{3}\left(2\sqrt{100-x^2} - \frac{x^2}{\sqrt{100-x^2}}\right) = \frac{\pi x}{3}\left(\frac{2\left(100-x^2\right) - x^2}{\sqrt{100-x^2}}\right)$

$=\frac{\pi x\left(200 - 3x^2\right)}{3\sqrt{100-x^2}}$

Q16aiii Let $\frac{dV}{dx} = \frac{\pi x\left(200-3x^2\right)}{3\sqrt{100-x^2}} = 0$, $200 - 3x^2 = 0$, $x = \frac{10\sqrt{2}}{\sqrt{3}}$

Major arc $= 2\pi r = 2\pi\times\frac{10\sqrt{2}}{\sqrt{3}} = \frac{20\pi\sqrt{2}}{\sqrt{3}}$

.: $10\theta = \frac{20\pi\sqrt{2}}{\sqrt{3}}$, $\theta = \frac{2\pi\sqrt{6}}{3}$ or $\theta = \frac{360\sqrt{6}}{3} = \left(120\sqrt{6}\right)°$

Q16bi No chance of winning: Same two numbers or two consecutive numbers. $(1,1)$, $(2,2)$, … , $(6,6)$

$(1,2)$, $(2,3)$, … , $(5,6)$ or in reverse order

Pr(no chance of winning before rolling the third) $= \frac{16}{36} = \frac{4}{9}$

Q16bii Difference of 5: $\Pr = \frac{2}{36}\times\frac{4}{6}$. Difference of 4: $\Pr = \frac{4}{36}\times\frac{3}{6}$

Difference of 3: $\Pr = \frac{6}{36}\times\frac{2}{6}$. Difference of 2: $\Pr = \frac{8}{36}\times\frac{1}{6}$

Pr(winning) $= \frac{40}{216} = \frac{5}{27}$

Q16ci $A_1 = 300000(1.04) - P$

$A_2 = \left(300000(1.04) - P\right)(1.04) - (1.05)P$

$= 300000(1.04)^2 - P\left[(1.04) + (1.05)\right]$

Q16cii $A_3 = \left(300000(1.04)^2 - P\left[(1.04) + (1.05)\right]\right)(1.04) - (1.05)^2 P$

$= 300000(1.04)^3 - P\left[(1.04)^2 + (1.04)(1.05) + (1.05)^2\right]$

Q16ciii $A_n = 300000(1.04)^n - P\left[\frac{(1.05)^n - (1.04)^n}{1.05 - 1.04}\right] > 0$

$0 < 3000(1.04)^n - P\left[(1.05)^n - (1.04)^n\right]$, $0 < \frac{3000}{P} - \left(\frac{1.05}{1.04}\right)^n + 1$

.: $\left(\frac{1.05}{1.04}\right)^n < 1 + \frac{3000}{P}$

Please inform mathline@itute.com re conceptual and/or mathematical errors.