



2018 VCAA Further Mathematics Exam 2 Solutions

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SECTION A - Core Data analysis

Q1a 3

Q1b 2

Q1c New castle-Sunderland, Liverpool

Q1d

Congestion level	City size	
	Small	Large
high	4	2
medium	4	2
low	8	3
Total	16	7

Q1e $\frac{4}{16} = 25\%$

Q1f Positively skewed

Q1g Upper fence = $Q_3 + 1.5 \times IQR = 39 + 1.5 \times 9 = 52.5$

Q2a Morning median 52%, evening median 56%

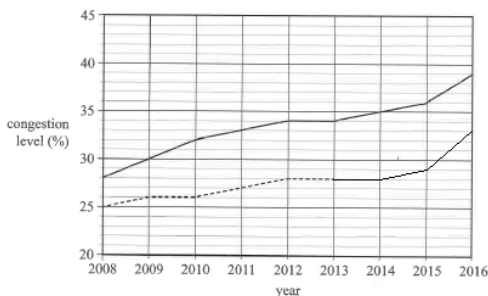
Q2b evening congestion level

Q2c evening congestion level = $8.48 + 0.922 \times 60 = 63.8$, i.e. 63.8%

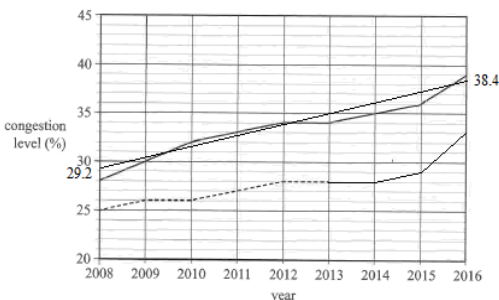
Q2d Residual = $50 - (8.48 + 0.922 \times 47) \approx -1.8$, i.e. -1.8%

Q2e $0.92^2 \approx 0.85 = 85\%$

Q3a



Q3bi



Q3bii 1.15% per year

Q3biii $-2280 + 1.15 \times \text{year} = 43$, year 2020

Q3c -1514

Q3d $\text{congestion level} = -1515 + 0.7667 \times \text{year}$

Q3e Compare the gradients: $m_{\text{Syd}} \approx 1.15$, $m_{\text{Mel}} \approx 0.7667$ $m_{\text{Syd}} > m_{\text{Mel}}$
In 2008 Sydney level is above Melbourne level, \therefore Sydney level will always exceed Melbourne level.

Recursion and financial modelling

Q4a \$12 000

Q4bi $V_1 = 1.0062 \times 12000 = 12074.40$

Q4bii $12000 \times 1.0062^4 \approx 12300.38$, \therefore after 4 months

Q4ci $\text{balance} = 12000 \times 1.0062^n$

Q4cii $n = 12 \times 3 = 36$

Q5a annual rate of depreciation = $1 - 0.85 = 0.15 = 15\%$

Q5b $\text{value} = 14000 \times 0.85^3 \times 0.914^5 \approx 5484.23$ dollars

Q6a $492800 \times R = 887.04 \times 26$, $R = 0.0468 = 4.68\%$

Q6bi TVM Solver $N = 12$, $I\% = 4.32$, $PV = -492800$,
 $FV = 480242.25$, $P/Y = 12$, $C/Y = 12$, \therefore $\text{PMT} = 2800.00$ dollars

Q6bii Using TVM Solver, $FV = 467131.13$

Total of 12 monthly payment in the second year
 $= 2800.00 \times 12 = 33600$

$480242.25 + \text{interest} = 467131.13 + 33600$

\therefore interest = 20488.88 dollars


SECTION B - Modules
Module 1: Matrices

Q1a \$2.87

Q1b 3×1

Q1c $M = \begin{bmatrix} 1 & 0 & 2 \end{bmatrix}$

Q2a $P_{2019} = \begin{bmatrix} 1.04 \times 2100 \\ 0.99 \times 1800 \\ 0.98 \times 1700 \end{bmatrix} = \begin{bmatrix} 2184 \\ 1782 \\ 1666 \end{bmatrix}$

Q2b $F = \begin{bmatrix} 1.04 & 0 & 0 \\ 0 & 0.99 & 0 \\ 0 & 0 & 0.98 \end{bmatrix}$

Q3a $0.1 \times 700 = 70$ km

Q3b $0.2 \times 700 + 0.1 \times 400 + 0.0 \times 200 + 0.2 \times 1400 = 460$

Q3c $S_1 = TS_0 = \begin{bmatrix} 460 \\ 390 \\ 360 \\ 1490 \end{bmatrix}$

Q3d $\frac{0.5 \times 1490}{1536} \approx 0.485 = 48.5\%$

Q3e

$$\begin{bmatrix} 0.2 & 0.1 & 0.0 & 0.2 \\ 0.1 & 0.1 & 0.0 & 0.2 \\ 0.2 & 0.1 & 0.2 & 0.1 \\ 0.5 & 0.7 & 0.8 & 0.5 \end{bmatrix}^{49} \approx \begin{bmatrix} 0.160 & 0.160 & 0.160 & 0.160 \\ 0.144 & 0.144 & 0.144 & 0.144 \\ 0.129 & 0.129 & 0.129 & 0.129 \\ 0.567 & 0.567 & 0.567 & 0.567 \end{bmatrix}$$

$$\begin{bmatrix} 0.2 & 0.1 & 0.0 & 0.2 \\ 0.1 & 0.1 & 0.0 & 0.2 \\ 0.2 & 0.1 & 0.2 & 0.1 \\ 0.5 & 0.7 & 0.8 & 0.5 \end{bmatrix}^{50} \approx \begin{bmatrix} 0.160 & 0.160 & 0.160 & 0.160 \\ 0.144 & 0.144 & 0.144 & 0.144 \\ 0.129 & 0.129 & 0.129 & 0.129 \\ 0.567 & 0.567 & 0.567 & 0.567 \end{bmatrix}$$

$0.567 = 56.7\%$

Q4a The length of the highway stays the same.

$\therefore k + 20 + 10 - 60 = 0, k = 30$

Q4b $M_1 = TM_0 + B = \begin{bmatrix} 470 \\ 410 \\ 360 \\ 1460 \end{bmatrix}, M_2 = TM_1 + B = \begin{bmatrix} 457 \\ 400 \\ 363 \\ 1480 \end{bmatrix}$

457 km are expected to be graded in 2022.

Module 2: Networks and decision mathematics

Q1ai 9

Q1aii 13

Q1b Maximum delivery = minimum cut = $3 + 2 + 2 = 7$

Q2a F

 Q2b Two extra edges, e.g. BD and GI

Q2c $F \rightarrow D \rightarrow C \rightarrow B \rightarrow A \rightarrow E \rightarrow H \rightarrow I \rightarrow J \rightarrow G$

Q3a Earliest starting time of 10 hours

 Q3b $BEGHJ$

 Q3c A and C

 Q3d End of E to start of J

Q4a 4

Q4b $P \rightarrow Q \rightarrow T \rightarrow S \rightarrow R \rightarrow Q \rightarrow P \rightarrow S \rightarrow T \rightarrow R \rightarrow P$
 $10 + 20 + 12 + 9 + 7 + 10 + 14 + 12 + 8 + 6 = 108$ km

Module 3: Geometry and measurement

Q1ai 6.8 cm

Q1b Area = $\pi \times 6.8 \times 20.4 + 2 \times \pi \times 3.4^2 = 508.4$ cm²

Q1ci $V = \frac{4}{3} \times \pi \times 3.4^3 \approx 164.6$ cm³

Q1cii $V = \pi \times 3.4^2 \times 20.4 \approx 740.9$ cm³

Q1ciii $V = 740.9 - 3 \times 164.6 \approx 247$ cm³

 Q2a Melbourne time: 10.30 pm to 12.00 midday (next day)
 \therefore travel time = 13 h 30 min

Q2bi Radius = $6400 \times \cos 11^\circ \approx 6282$ km

Q2bii Distance $\approx \frac{123 - 107}{360} \times 2 \times \pi \times 6282 \approx 1754$ km

Q3a $AB = \sqrt{4.1^2 + (6.4 + 6.4 + 5.5)^2} \approx 18.8$ m

Q3b Distance $\approx \sqrt{18.8^2 + 2.5^2} \approx 19$ m

Q3ci $\frac{\sin \angle AQP}{20.7} = \frac{\sin 23.5^\circ}{10.4}, \angle AQP \approx 52.5^\circ$ or $180^\circ - 52.5^\circ = 127.5^\circ$

Q3cii $\angle APQ \approx (180 - 127.5 - 23.5)^\circ \approx 29^\circ$

$\frac{x}{\sin 29^\circ} \approx \frac{10.4}{\sin 23.5^\circ}, x \approx 13$ m



Module 4: Graphs and relations

Q1a 2nd Nov

Q1b 16th Nov and 17th Nov

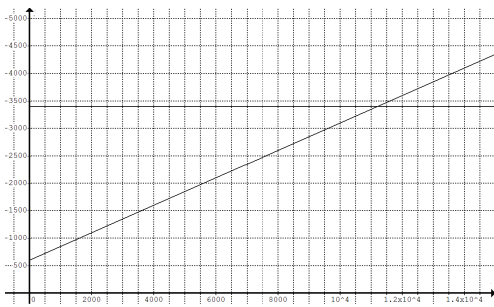
Q2a $567 = M \times 20$, $M = 28.35$

Q2b *weight in grams* = $28.35 \times 0.2 = 5.67$

Q2c $\frac{12474}{55} = 28.35 \times \textit{weight in ounces}$

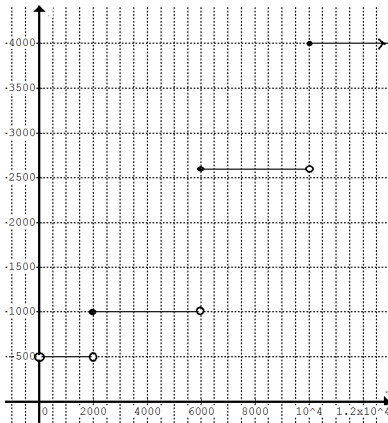
weight in ounces = $\frac{12474}{55 \times 28.35} = 8$

Q3a

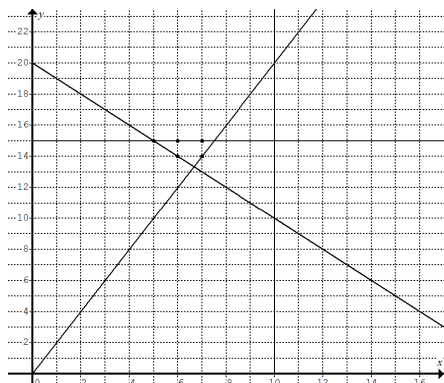


Q3b $600 + 0.25x = 3400$, $x = 11200$ dollars

Q3c



Q4a A dot is used instead of a cross to mark the five integer points.



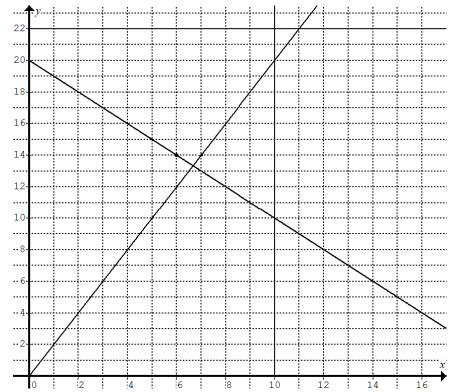
Q4b

$(6, 15): 130 \times 6 + 110 \times 15 = 2430$

$(7, 14): 130 \times 7 + 110 \times 14 = 2450$

$(7, 15): 130 \times 7 + 110 \times 15 = 2560$ dollars (max cost)

Q4c



$(6, 14): 130 \times 6 + 140 \times 14 = 2740$ dollars (min cost)

Please inform mathline@itute.com re conceptual and/or mathematical errors