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2018 VCAA Mathematical Methods Exam 1 Solutions

Q1a
$$\frac{dy}{dx}=3\left(-3x^3+x^2-64\right)^2\left(-9x^2+2x\right)=-3\left(3x^3-x^2+64\right)^2\left(9x^2-2x\right)$$

Q1b $f'(x)=\dfrac{\cos(x)e^x-e^x(-\sin(x))}{\cos^2(x)}=\dfrac{e^x(\cos(x)+\sin(x))}{\cos^2(x)}$

$$f'(\pi)=\frac{e^\pi(\cos(\pi)+\sin(\pi))}{\cos^2(\pi)}=\frac{e^\pi(-1)}{(-1)^2}=-e^\pi$$

Q2 $f(x)=\int\frac{1}{2}\left(1-\frac{1}{x-1}\right)dx=\frac{1}{2}x-\frac{1}{2}\log_e(x-1)+C$

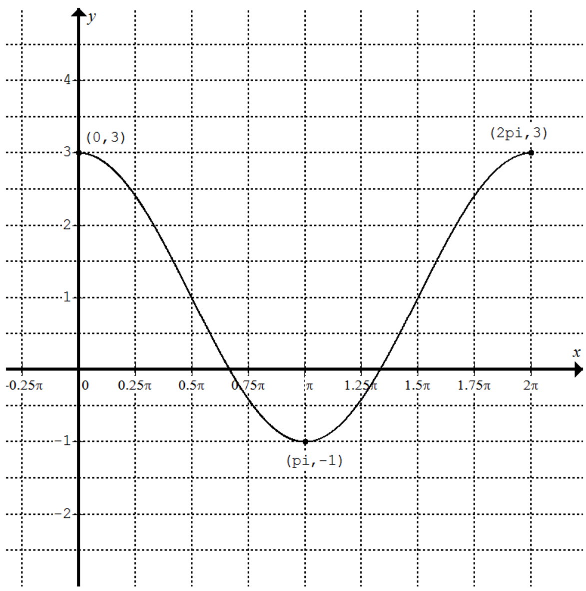
$f(2)=\frac{1}{2}(2)-\frac{1}{2}\log_e(1)+C=0$, $C=-1$

$f(x)=\frac{1}{2}x-\frac{1}{2}\log_e(x-1)-1$ for $x\in(1,\infty)$

Q3a $2\cos(x)+1=0$ and $0\le x\le 2\pi$

$\cos(x)=-\frac{1}{2}$, $x=\frac{\pi}{3},\frac{4\pi}{3}$

Q3b



Q4a $\Pr(X>6)=0.5$

Q4b $\Pr(X>7)=\Pr(X<5)=\Pr\left(Z<\frac{5-6}{2}\right)=\Pr\left(Z<-\frac{1}{2}\right)$,

$\therefore\ b=-\frac{1}{2}$

Q5a $x=\frac{1}{(y-2)^2}$, $y=\frac{1}{\sqrt{x}}+2$, $f^{-1}(x)=\frac{1}{\sqrt{x}}+2$, domain: $(0,\infty)$

Q6a $\Pr(\text{black})=\frac{1}{2}\Pr(\text{black from box 1})+\frac{1}{2}\Pr(\text{black from box 2})$

$=\frac{1}{2}\times 1+\frac{1}{2}\times\frac{2}{4}=\frac{3}{4}$. Note: Same probability as drawing a black from a box containing 6 blacks and 2 whites.

Q6b $\Pr(\text{Box 1}\mid\text{black})=\dfrac{\Pr(\text{Box 1}\cap\text{black})}{\Pr(\text{black})}=\dfrac{\frac{1}{2}}{\frac{3}{4}}=\dfrac{2}{3}$

Q7a $L_{OP}=\sqrt{x^2+y^2}=\sqrt{x^2+(2x-4)^2}=\sqrt{5x^2-16x+16}$

Let $\dfrac{dL_{OP}}{dx}=\dfrac{10x-16}{2\sqrt{5x^2-16x+16}}=0$, $x=\frac{8}{5}$, $y=2\left(\frac{8}{5}\right)-4=-\frac{4}{5}$

$\therefore\ P\left(\frac{8}{5},-\frac{4}{5}\right)$

Q7b $L_{OP}=\sqrt{x^2+y^2}=\sqrt{\left(\frac{8}{5}\right)^2+\left(-\frac{4}{5}\right)^2}=\frac{1}{5}\sqrt{80}=\frac{4\sqrt{5}}{5}$

Q8a $f'(x)=kx^2e^{kx}+2xe^{kx}=xe^{kx}(kx+2)$

Q8b $x^2e^{kx}=xe^{kx}(kx+2)$, $x^2=x(kx+2)$, $x^2=kx^2+2x$,
$(k-1)x^2+2x=0$, $\therefore$ one solution (one point of intersection at $x=0$) when $k-1=0$, $\therefore\ k=1$

Q8c $A=\int_0^2\left(x^2e^{kx}+\frac{2xe^{kx}}{k}\right)dx$

Q8d $f'(x)=xe^{kx}(kx+2)$, $\therefore\ x^2e^{kx}+\frac{2xe^{kx}}{k}=\frac{1}{k}f'(x)$

$\therefore\ A=\int_0^2\frac{1}{k}f'(x)dx=\frac{1}{k}\left[f(x)\right]_0^2=\frac{1}{k}\times 2^2e^{2k}=\frac{16}{k}$

Let $\frac{1}{k}\times 2^2e^{2k}=\frac{16}{k}$, $e^{2k}=4$, $e^k=2$, $k=\log_e 2$

Q9ai $\int_{n\pi}^{(n+1)\pi}(x\sin(x))dx=\left[\sin(x)-x\cos(x)\right]_{n\pi}^{(n+1)\pi}$
$=\sin((n+1)\pi)-(n+1)\pi\cos((n+1)\pi)-\sin(n\pi)+n\pi\cos(n\pi)$
$=0-(n+1)\pi(-1)-0+n\pi(1)$
$=(2n+1)\pi$ when n is a positive even integer

Q9aii $\int_{n\pi}^{(n+1)\pi}(x\sin(x))dx$
$=\sin((n+1)\pi)-(n+1)\pi\cos((n+1)\pi)-\sin(n\pi)+n\pi\cos(n\pi)$
$=0-(n+1)\pi(1)-0+n\pi(-1)$
$=-(2n+1)\pi$ when n is a positive odd integer

Q9b

$\frac{dy}{dx} = \sin(x) + x\cos(x) = \sin\left(-\frac{5\pi}{2}\right) - \frac{5\pi}{2}\cos\left(-\frac{5\pi}{2}\right) = -1$ at

$x = -\frac{5\pi}{2}$

Equation of the tangent: $y - \frac{5\pi}{2} = -1\left(x + \frac{5\pi}{2}\right)$, $y = -x$

Q9c $y = (3\pi - x)\sin(x) = (x - 3\pi)\sin(x - 3\pi)$, .: $a = 3\pi$

Q9d

ℓ_1 is the translation of the tangent in part b to the right by 3π

ℓ_1 is $y = -(x - 3\pi)$ and ℓ_2 is $y = x - 3\pi$, y-intercept of ℓ_1 is 3π

Shaded area

$$= 2\left(\frac{1}{2}(3\pi)^2 - \left(\int_0^{\pi}(x\sin(x))dx - \int_{\pi}^{2\pi}(x\sin(x))dx + \int_{2\pi}^{3\pi}(x\sin(x))dx\right)\right)$$

$$= 2\left(\frac{1}{2}(3\pi)^2 - (2\times 0 + 1)\pi - (2\times 1 + 1)\pi - (2\times 2 + 1)\pi\right)$$

$$= 9\pi(\pi - 2)$$

Please inform mathline@itute.com re conceptual and/or mathematical errors