

**2018 Year 11 math topic test: Circular functions, symmetry and identities** © itute 2018

Do not use CAS or calculators in this test

Q1 Find the exact value/s of

a. $\cos(-225^\circ)$ 1 mark

b. $-\cos^2(111^\circ) - \sin^2(111^\circ)$ 1 mark

c. $\cot\left(\frac{-3\pi}{2}\right)$ 1 mark

d. $\cos^2(195^\circ) - \sin^2(195^\circ)$ 2 marks

e. $\cos\left(\frac{-7\pi}{12}\right)\sin\left(\frac{7\pi}{12}\right)$ 2 marks

f. $\cos(12^\circ)\cos(33^\circ) - \sin(-12^\circ)\sin(-33^\circ)$ 2 marks

g. $\tan\left(\frac{5\pi}{12}\right) - 1$ 2 marks

h. $\tan(\theta)$ if $\operatorname{cosec}(\theta) = -3$ 2 marks

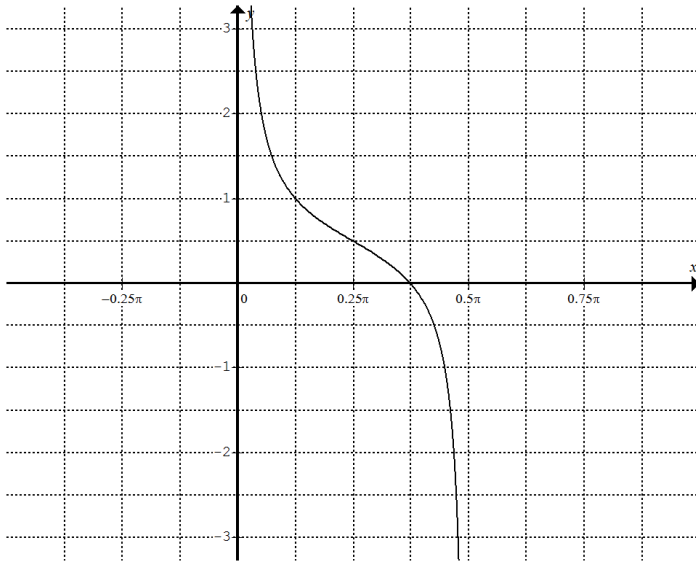


Q2 The graph of $f(x) = \frac{1}{2} \left(1 - \tan \left(2x - \frac{\pi}{2} \right) \right)$ in the interval $\left(0, \frac{\pi}{2} \right)$ is shown below.

On the same axes, sketch the graph of $g(x) = 2 \left(f \left(x + \frac{\pi}{4} \right) - \frac{1}{2} \right)$ in the interval $\left[0, \frac{\pi}{2} \right]$.

Show and label axes intercepts and end points with coordinates, and asymptote/s with equation/s.

6 marks



Q3 Consider $f(x) = \sqrt{3} \cos(x) - \sin(x)$.

a. $f(x)$ can be expressed in the form $a \sin(x + \delta)$ where a and δ are real numbers. Find an exact value for each of a and δ .

3 marks

b. $f(x)$ can also be expressed in the form $a \cos(x + \epsilon)$ where a and ϵ are real numbers. Find the smallest positive exact value of ϵ .

2 marks

c. Hence solve $f(x) = -\sqrt{2}$ for $x \in [0, 2\pi]$.

2 marks



Q4 Solve each of the following equations for x .

a. $2\cos^2(x) - \cos(2x) = 1$ where $x \in [0, 2\pi]$ 2 marks

b. $\tan\left(2\left(x + \frac{\pi}{8}\right)\right) - \tan\left(3\left(x - \frac{\pi}{12}\right)\right) = 0$ where $x \in [0, 2\pi]$ 3 marks

c. $\sec^2(x) - \operatorname{cosec}^2(x) = 0$ where $x \in [0, 2\pi]$ 3 marks

d. $\cos(x - \delta) + \cos(x + \delta) = 0$ where $x \in [0, 2\pi]$ and $\delta \in R$ 3 marks

Q5 Given $x = \sec(t)$ and $y = \cot(t)$, express y in terms of x . 4 marks



Q6 Prove the following statements.

a.
$$\frac{\tan(x) - \cot(x)}{\sec(x) - \operatorname{cosec}(x)} = \sin(x) + \cos(x)$$

3 marks

b.
$$\sin(u + v) + \cos(u - v) = (\sin(u) + \cos(u))(\sin(v) + \cos(v))$$

3 marks

c.
$$\frac{\cos(x) + \sin(x)}{\cos(x) - \sin(x)} = \sec(2x) + \tan(2x)$$

3 marks