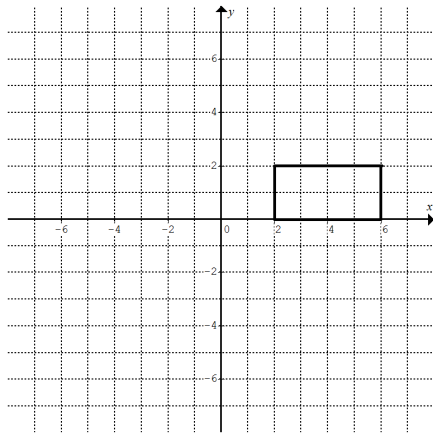




**2018 Year 11 math topic test: Linear transformations of the plane** © itute 2018

Q1 Dilate the rectangle in the following diagram from the  $x$ -axis by a factor of 2 and from the  $y$ -axis by a factor of  $\frac{1}{2}$ , and then reflect in the  $x$ -axis.



a. Sketch accurately the resulting figure on the same diagram. 3 marks

b. Write down a  $2 \times 2$  matrix to represent the transformations. 2 marks

c. Use the matrix in part b to find the images of  $(2, 2)$  and  $(6, 0)$ . 2 marks

Q2 Consider matrix  $\begin{bmatrix} -\frac{3}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$ .

a. State the transformations of the plane represented by the matrix. 3 marks

b. A square has an area of 4 units. Give the name of the shape of the image after the transformations. 1 mark

c. Calculate the area of the image from its dimensions. 1 mark



Q3 Use matrices to show that rotation of a point about the origin by  $180^\circ$  gives the same image as reflection in the line  $y = x$  and reflection in the line  $y = -x$  in either order.

4 marks

Q4 Line  $\ell_1$ , given by  $y = \sqrt{3}x$ , is reflected in the line  $y = \frac{x}{\sqrt{3}}$ . The image is line  $\ell_2$ .

a. Find the matrix which represents reflection in the line  $y = \frac{x}{\sqrt{3}}$ .

4 marks

b. Point  $P(x, y)$  is on line  $\ell_1$ . Find the image of point  $P$  after reflecting in the line  $y = \frac{x}{\sqrt{3}}$ .

3 marks

c. Find the equation of line  $\ell_2$ .

3 marks



Q5 Consider the linear transformations of the plane given by  $(x, y) \rightarrow \left( -\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}, \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} \right)$ .

a. Write down the  $2 \times 2$  matrix which represents the transformations. 2 marks

b. Show that the  $2 \times 2$  matrix in part a represents the following transformations:  
Reflection in the line  $y = x$ , and then rotation about the origin by  $45^\circ$  anticlockwise. 3 marks

c. Find the image of  $(x, y)$  if rotation about the origin by  $45^\circ$  anticlockwise is performed first, followed by reflection in the line  $y = x$ . 3 marks

d. Find the matrix which represents the transformations  $\left( -\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}, \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} \right) \rightarrow (x, y)$ . 3 marks



Q6 The graph of  $y = x^2 - 1$  is translated in the negative  $x$  direction by 1 unit and in the positive  $y$  direction by 2 units, and then reflected in the  $x$ -axis. Let  $(x, y)$  be a point on the graph and  $(x', y')$  be its image after the transformations.

a. Write a matrix equation showing the relation between  $(x', y')$  and  $(x, y)$ . 3 marks

b. Find  $x'$  and  $y'$  in terms of  $x$  and  $y$ . 3 marks

c. Hence find the image of the graph of  $y = x^2 - 1$  after the transformations. 3 marks

Q7 Consider the circle given by  $(x - 7)^2 + (y + 9)^2 = \frac{1}{3\pi}$ .

The  $x, y$  plane is transformed according to the rule  $(x, y) \rightarrow \left(-kx + y, x + \frac{2y}{k}\right)$  and  $k \in R$ .

Find the area of the image of the circle after the transformations. 4 marks