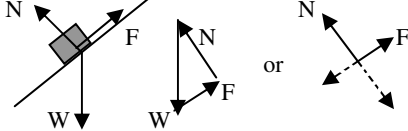


## Physics (2017-2021) Summary sheets © 2018 itute

According to Newton, space (length), time and mass are **absolute**, i.e. they remain the same irrespective of the observers.

**Newton's first law:** Objects have **inertia**, i.e. a stationary object remains stationary, or a moving object keeps on moving at the same speed in the same direction, if there is no net force acting on it. **Newton's second law:** Acceleration of an object is directly proportional to and in the same direction as the net force on it, and inversely proportional to its mass.  $a = F_{net}/m$ . **Newton's third law:** When object A exerts a force on object B, B exerts a force of the same magnitude in the opposite direction on A.

**Net force** is determined by vector addition. In one dimension: by addition of directed numbers. In two dimensions: by placing vectors head to tail or by resolving each vector into two perpendicular components. E.g. net force on an object at rest (or sliding at const speed) on an inclined plane is zero.



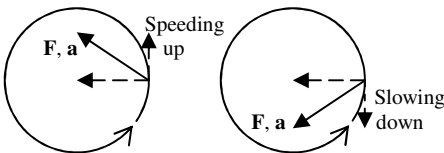
**Uniform (constant speed v) circular motion:**

$f = \frac{1}{T}$ ,  $v = \frac{2\pi r}{T}$  or  $2\pi r f$ , direction of motion is given by velocity vector that is tangential to the circular path;

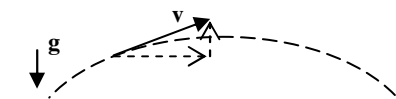
magnitude of acceleration is  $a = \frac{v^2}{r}$  or  $a = \frac{4\pi^2 r}{T^2}$  or

$a = 4\pi^2 r f^2$ , and direction of acceleration is always towards the centre of circle,  $\therefore$  **centripetal acceleration**. Both velocity and acceleration in uniform circular motion are not constant because their directions are changing continuously. They are always perpendicular to each other. A net force towards the centre of the circle (**centripetal force**) is required to keep an object in uniform circular motion,  $F = ma$ .

**Non-uniform circular motion:** Besides the centripetal force, a tangential force is also required to speed up or to slow down the object. Hence the net force and the acceleration are no longer towards the centre of the circular motion.



**Projectile motion:** Two-dimensional motion under a constant force (force of gravity or weight).



**Horizontal component** of velocity vector remains constant throughout motion. **Vertical component** of velocity vector is affected by gravity and has constant acceleration g downwards. Let V be the speed of projection at angle  $\theta$  to the horizontal.

For hori. comp:  $a = 0$ ,  $v = u = V \cos \theta$ ,  $s = ut$

For vert. comp: the five equations for rectilinear motions under constant acceleration are applicable,

$v = u + at$ ,  $s = \frac{1}{2}(u + v)t$ ,  $s = ut + \frac{1}{2}at^2$ ,  $s = vt - \frac{1}{2}at^2$ ,

$v^2 = u^2 + 2as$ , where  $u = V \sin \theta$  is the initial velocity,

$v$  final vel,  $a = -g$  acceleration,  $s$  displacement from the initial position at time  $t$ . Up is chosen as +ve.

**Impulse = change in momentum,  $I = \Delta p$ .**

$F\Delta t = mv - mu$ . **Conservation of momentum:** In collisions between objects, total momentum before = total momentum during = total momentum after collision,

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$ .

When one object gains momentum, the other loses momentum by the same amount, the total remains constant.

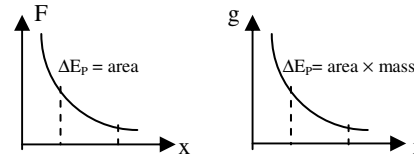
$\Delta p_2 = -\Delta p_1$ , i.e.  $I_2 = -I_1$ .

**Work** is done by a system on another system during energy transfer in which the former exerts a force on the latter.  $W = Fs = \Delta E$ .

**Change in kinetic energy** of an object results from work done by net force.  $W = \Delta E_K$  i.e.  $F_{net} s = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ .

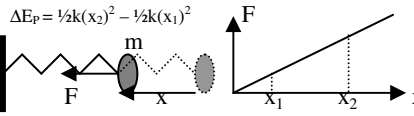
When an object moves in a gravitational field, e.g. in projectile motion, kinetic energy changes to **gravitational potential energy** and vice versa. The total energy remains constant during its flight.  $E_{K1} + E_{P1} = E_{K2} + E_{P2}$ .

At Earth's surface,  $E_P = mgh$ ,  $\Delta E_P = mgh_2 - mgh_1$ . If an object moves a long distance away from (or towards) the earth, gravitational field cannot be considered constant,  $\Delta E_P$  is given by area under force-distance graph or field-distance graph:



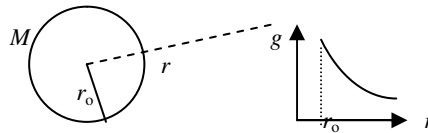
**Hooke's law:**  $F = kx$ . When an object interacts with a spring that obeys Hooke's law, kinetic energy is changed to **elastic potential energy** and vice versa. The total energy remains constant during the interaction.  $E_{K1} + E_{P1} = E_{K2} + E_{P2}$  where  $E_P = \frac{1}{2}kx^2$ .

Area under force-distance graph gives

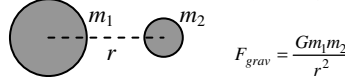


**Elastic collision** between objects: the total kinetic energy of objects before and after collision remains the same. During collision some kinetic energy is changed to elastic potential energy and all elastic potential energy is changed back to kinetic energy at the end of collision. For **inelastic collision**, total kinetic energy after collision is less than total before, because some kinetic energy is changed to other forms of energy as well, such as sound and heat.

**Universal gravitational field:**  $g = \frac{GM}{r^2}$



**Gravitational force** between any two objects:



**Planetary and satellite motions:** Planets around the sun move in its gravitational field;  $\therefore a = g$ , they are in free fall (also true for satellites around the earth),

i.e.  $\frac{v^2}{r} = \frac{GM}{r^2}$  or  $\frac{4\pi^2 r}{T^2} = \frac{GM}{r^2}$ , hence

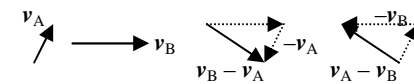
$v^2 r = GM$  (constant) or  $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$  (const).

$\therefore v_a^2 r_a = v_b^2 r_b$  or  $\frac{r_a^3}{T_a^2} = \frac{r_b^3}{T_b^2}$ .

**Inertial frame of reference:** A frame of reference in which Newton's first law is valid. Frames of reference that are stationary or moving at constant velocity are inertial frames. Accelerating frames are **non-inertial**.

**Relative motion:** When object A moves at velocity  $v_A$  and object B moves at velocity  $v_B$  as determined by the same observer (usually but not necessarily taken as stationary), then velocity of B relative to A is  $v_B - v_A$  and velocity of A relative to B is  $v_A - v_B$ .

In 1-D: by subtraction of directed numbers. In 2-D:



**The postulates of Einstein's special theory of relativity**

- (1) The laws of physics are the same in all inertial frames of reference.
- (2) The speed of light has a constant value for all observers regardless of their motion or the motion of the source.

Postulate (1) was an extension of the Newtonian model to include not only the laws of mechanics but also those of the rest of physics, including electricity and magnetism.

Postulate (2) was completely different from that of Newton. The notion of relative velocity was discarded and replaced with the idea that the speed of light in vacuum is always the same, no matter what the speed of the observer or the source. Hence the existence of the ether as the absolute frame of reference was no longer required.

Time interval between two events that is measured in the frame of reference where the two events occur at the same point in space is known as **proper time**  $t_0$ .

If it is measured in a frame of reference travelling at speed  $v$  relative to the first one, the time interval  $t$  between the two events will be longer. This is known as **time dilation**.

$$\text{Time dilation: } t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \text{ i.e. } t = \gamma t_0, \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

For muons created in the lab on Earth, their half-lives measured by an observer in the lab are the proper times. Muons travelling towards Earth at speed  $v$  relative to the Earth observer will have dilated half-lives.  $\therefore$  muons can reach Earth even though their half-lives would suggest that they should decay in the outer atmosphere.

If an object (or two objects) is at rest relative to an observer, the length of the object (or distance between two objects) measured by the observer is known as its **proper length**  $L_0$ .

If the object is moving at speed  $v$  relative to the observer (or the observer is moving at speed  $v$  relative to the object), the length of the object measured by the observer will be shorter. This is known as **length contraction**.

$$\text{Length contraction: } L = L_0 \sqrt{1 - \frac{v^2}{c^2}}, \text{ i.e. } L = \frac{L_0}{\gamma}$$

**Total mass-energy** is given by  $E_{total} = E_k + E_0 = \gamma mc^2$  where  $E_0 = mc^2$ ,  $\therefore E_k = (\gamma - 1)mc^2$

Matter is converted to energy by nuclear fusion in the Sun, which leads to its mass decreasing and resulting in the emission of electromagnetic radiation.

**Electric current** through a component is measured with an **ammeter** connected in series with it.  $I = \frac{Q}{t}$ ,  $Q = It$ .

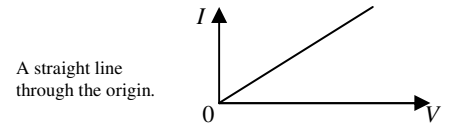
**Electric potential**  $V$  (v) at a point is the amount of electric potential energy  $E$  (J) possessed by each unit of charge at that point.  $V = \frac{E}{Q}$ ,  $E = VQ$ .

**Electric potential difference**, also denoted as  $V$  and measured in v, is the difference in potential between two points. When current flows from high to low potential, electric potential energy of the charges changes to other forms of energy. Amount of energy change is also given by

$E = VQ$  where  $V$  is the potential difference measured with a **voltmeter** connected to the two points.  $E = VQ = VIt$ ,  $E = Pt$ , **power**  $P = VI$ .

**Resistance**  $R$  of a conductor is a measure of the ability of the conductor to resist the flow of electric current and is defined as the ratio of  $V$  to  $I$ .  $R = \frac{V}{I}$ .

**Ohm's law** states that for some conductors the resistance stays **constant** when potential difference and current vary. Conductors that obey Ohm's law are called **ohmic conductors (resistors)** and have the following  $I$ - $V$  characteristics.

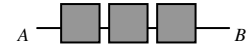


**Components connected in series**

$$I = I_1 = I_2 = I_3 = \dots \quad V_{AB} = V_1 + V_2 + V_3 + \dots$$

$R_T = R_1 + R_2 + R_3 + \dots$  remains constant if components

are ohmic resistors. Also  $R_T = \frac{V_{AB}}{I}$ .



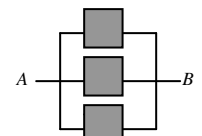
**Components connected in parallel**

$$V_{AB} = V_1 = V_2 = V_3 = \dots$$

$$I = I_A = I_1 + I_2 + I_3 + \dots = I_B$$

$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots}$  remains constant for ohmic resistors.

Also  $R_T = \frac{V_{AB}}{I}$ .

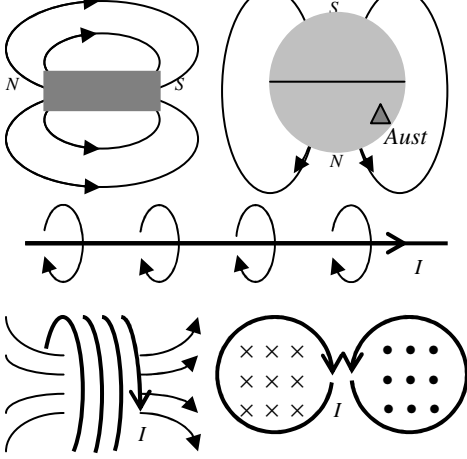


**Power in series and parallel circuits:** Total power consumption in parallel or series connection is the sum of the individual power of the components.

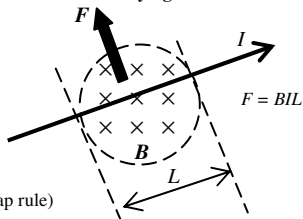
$$P_T = P_1 + P_2 + P_3 + \dots$$

$$\text{or } P_T = V_{AB} I \text{ or } P_T = I^2 R_T \text{ or } P_T = \frac{V_{AB}^2}{R_T}$$

**Magnetic field B** Unit: tesla (T); or weber per square metre ( $\text{wbm}^{-2}$ ):

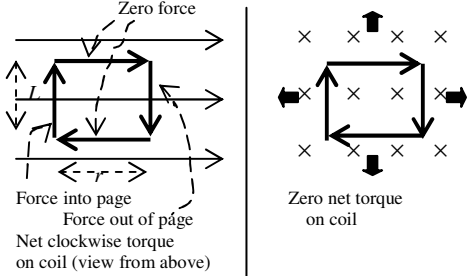


Use right-hand grip rule to find direction of  $B$ .  
**Magnetic force on current-carrying wire:**



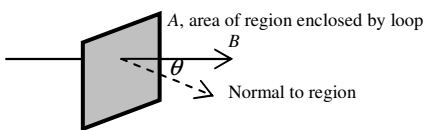
Direction of  $F$   
 (Right-hand slap rule)

**Magnetic force on current-carrying coil of  $n$  loops:**



Refer to above diagrams, the two forces (each  $F = nBIL$  in and out of the page) exert a turning effect called torque ( $\tau = rF$ ) on the coil causing it to speed up its rotation in the first quarter turn. In the second quarter turn the torque is in the opposite direction causing the coil to slow down. In a **simple DC motor**, the direction of the coil current is reversed with a **split-ring commutator** every  $180^\circ$  turn when the net torque is zero. Hence the torque on the coil remains in the same direction, allowing the coil in the motor to keep turning in the same direction.

**Magnetic flux  $\Phi = BA \cos \theta$ .** Unit: weber (wb)

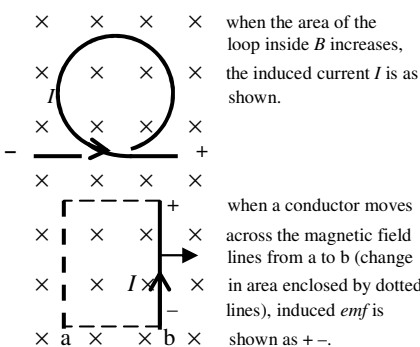


**Electromagnetic induction** is the generation of electricity by changing magnetic flux. The generated current is called **induced current  $I$** ; the generated voltage is called **induced emf  $\xi$** .

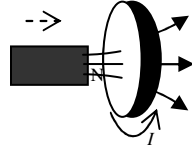
Magnitude of  $\xi_{av} = \frac{\Delta \Phi}{\Delta t}$ ,  $I = \frac{\xi}{R}$  where  $\Delta t$  time taken for the change,  $R$  resistance of the loop.

**Ways to induce emf or current:**

Since  $\Phi = BA \cos \theta$ , emf can be induced  
 (1) by changing  $A$  to change  $\Phi$

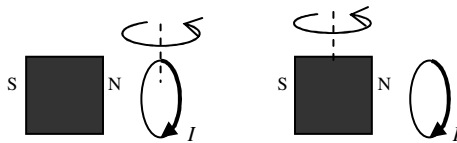


(2) by changing  $B$  to change  $\Phi$



when magnet moves closer to the loop, magnetic field increases and the induced current  $I$  is as shown.

(3) by changing  $\theta$  (either by rotating the loop or the magnet) to change  $\Phi$

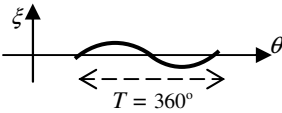


Direction of induced current and polarity of output terminals of a generator are determined by **Lenz's Law**: an induced current in a conducting loop flows in a direction such that the magnetic field of the induced current *opposes* the change in magnetic flux that produces it. The terminal that the induced current flows to is +, the other -.

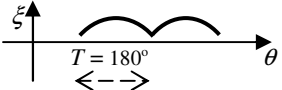
**Faraday's Law:**  $|\xi_{av}| = n \frac{\Delta \Phi}{\Delta t}$  where  $n$  is the number of loops in the coil through which the flux passes.

$$\xi_{av} \propto n, \xi_{av} \propto \Delta \Phi, \xi_{av} \propto \frac{1}{\Delta t}$$

**AC generator (alternators):** Alternating emf induced by a rotating conducting coil (loop area  $A$ ) in a magnetic field  $B$  is made accessible with **slip-rings** connected to the terminals of the coil, and the external circuit is connected to the rings via conducting brushes.



If the slip-rings are replaced by a **split-ring commutator**, the device is a **DC generator**.



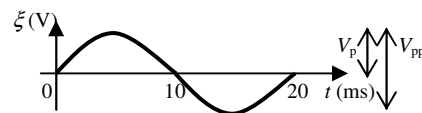
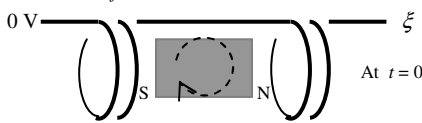
The amplitude of the AC emf is called the peak voltage  $V_p$ .  
 $V_p \propto n, V_p \propto B, V_p \propto A, V_p \propto f$

**DC motor has the same construction, and can be used as a DC generator by turning the coil mechanically.**

Alternating emf can also be induced by rotating a permanent magnet (or electromagnet) beside a coil. External circuit is connected directly to the terminals of the coil and no slip rings are required.

**AC power supply** delivered to homes and offices are generated by rotating an electromagnet between two connected coils at

$$f = 50 \text{ Hz } (T = \frac{1}{f} = 0.02 \text{ s} = 20 \text{ ms})$$



Peak voltage  $V_p = 340 \text{ V}$ , peak-to-peak voltage  $V_{pp} = 680 \text{ V}$ ,

$$\text{root-mean-square voltage } V_{rms} = \frac{V_p}{\sqrt{2}} = 240 \text{ V}$$

An AC supply of  $V_{rms} = 240 \text{ V}$  provides the same power as a DC supply of constant  $V = 240 \text{ V}$ .

$$I_p = \frac{V_p}{R}, I_{pp} = \frac{V_{pp}}{R}, I_{rms} = \frac{V_{rms}}{R}, I_{rms} = \frac{I_p}{\sqrt{2}}$$

$$P = P_{av} = \frac{V_p^2}{2R} = \frac{1}{2} I_p^2 R = \frac{V_p I_p}{2} \text{ or}$$

$$P = P_{av} = \frac{V_{rms}^2}{R} = I_{rms}^2 R = V_{rms} I_{rms}$$

In a power station the generator **always rotates at the same rate**. If the power consumption by homes and offices is higher (lower), more (less) energy is required to maintain the same rotation rate, 50 Hz in Australia.

**Transformer:** An electrical device that is used to change the voltage of an AC power supply without changing the power to be delivered.

**Working of a transformer:** Alternating current at the primary (input) coil produces an alternating  $B$  inside the soft iron core. The secondary (output) coil is linked to the primary through the core, a changing  $B$  in the core results in a changing  $\Phi$  in the secondary coil. According to Faraday's Law an emf is induced in the secondary coil (output).

Step-up transformer  $N_s > N_p$ ; step-down  $N_s < N_p$ .

For an ideal (100% efficiency) transformer,  $P_s = P_p$ ,

$$V_s I_s = V_p I_p, \frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}$$

**Power loss  $P_{loss}$  and voltage drop  $V_{drop}$**  occur when electricity is transmitted over a long distance by transmission lines.

$$V_A \updownarrow \text{Transmission lines current } I, \text{ resistance } R \updownarrow V_B < V_A$$

$$V_{drop} = V_A - V_B = IR; P_{loss} = I^2 R = V_{drop} I = \frac{V_{drop}^2}{R}$$

$P_{loss} \propto I^2$ , power loss is greatly reduced by lowering  $I$  in the lines; this can be achieved by increasing the voltage for transmission in order to deliver the same power,  $\therefore$  step-up transformer is used at the power station end. At the consumer end, step-down transformer is used to reduce voltage to 240 V.

**Load curve** is a graph showing the demand of electric power over a time period. Area under the curve represents the total consumption of electrical energy during the period.  
 $E = P \Delta t$

**Kilowatt-hour (kwh)** is a unit of electrical energy.

$$1 \text{ kwh} = 3.6 \text{ MJ}$$

**Comparison of gravitational, electric and magnetic fields**

The fields are used to explain the cause of motion of objects when there is no apparent contact, i.e. force at a distance.

$$F_{grav} = \frac{GMm}{r^2}, g = \frac{GM}{r^2}; F_{elec} = \frac{kQq}{r^2}, E = \frac{kQ}{r^2}$$

Gravitational field of mass  $M$  and electric field of charge  $Q$  both follow inverse square law. Magnetic field  $B$  does not. Electric force and magnetic force can be attractive and repulsive. Gravitational force is always attractive. Fields are vector quantities. Grav. field always points towards a mass. Elec. field points away from a positive charge towards a negative charge. Magn. field points away from a north magnetic pole towards a south magnetic pole.

**Uniform field** means the field is constant in space.

**Static field** means the field is constant in time.

**Waves** A wave **pulse** is generated when a stretched spring is given a shake at one end. This wave pulse travels along the spring to the other end.

The spring is given a certain amount of energy during the shake. This amount of energy exists in the spring and is carried along the spring by the wave pulse.

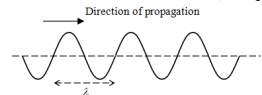
The spring is the medium for the wave pulse (energy) to travel along. Particles of the medium are displaced while the pulse is passing and they returned to their original positions after the pulse is through. They do not travel with the pulse. The transfer of energy from one place to another does not involve the net transfer of any material of the medium.

When the spring is shaken perpendicular to the length of the spring, the motion of a particle in the spring and the motion of the pulse are perpendicular to each other. This type of waves is categorised as **transverse waves**.

Wave pulses can also be generated by shaking a stretched spring along the direction of its length.

The motion of a particle in the wave is parallel to the motion of the pulse. This type of waves is categorised as **longitudinal waves**. Again the particles do not travel with the pulse. They are displaced when the pulse is passing through them.

If the shaking of the spring is done repeatedly, a periodic travelling wave is formed in the medium (the spring).



A 'full' shake produces a cycle of the periodic wave. The time interval for generating a cycle is the **period  $T$**  of the wave. The length of a cycle of the wave is called its **wavelength  $\lambda$** . The number of cycles generated in a unit time (second) is the frequency  $f$  of the periodic wave.

Frequency and period of a wave are related,  $f = \frac{1}{T}$ .

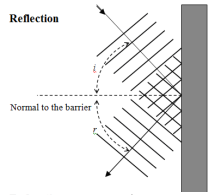
The highest point of a wave is the crest, and the lowest point the trough. Half way between the two is the equilibrium position where the spring is. The distance between a crest (or trough) and the equilibrium position is the amplitude of a wave.

A travelling wave moves a distance  $\lambda$  (the wavelength) during a time interval  $T$  (the period). Hence the speed of the wave  $v$  is  $v = \frac{\lambda}{T}$  or  $v = f \lambda$ .

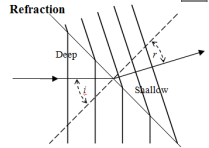
**Phase** Two particles in a wave are in **phase** when they move in the **same direction** and at the **same speed**. The distance between two consecutive particles in phase equals the wavelength.

Two waves of the same frequency are in phase when they vibrate the same way at the same place and at the same time.

**Surface water waves** are transverse waves because the motion of water particles is perpendicular to the direction of propagation of the waves. The highest points are called **crests** and the lowest points are **troughs**.

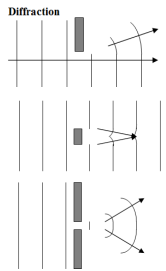


Wavelength, frequency and speed remain the same after reflection. Angles of incidence and reflection are equal,  $\angle i = \angle r$ . This is known as the **law of reflection**.



The wave changes its speed and direction when the depth of water changes. Frequency is the same in both regions.

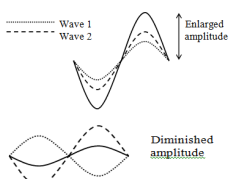
$$\lambda_d > \lambda_s, v_d > v_s, \angle i > \angle r, \frac{v_d}{v_s} = \frac{f\lambda_d}{f\lambda_s} = \frac{\lambda_d}{\lambda_s}$$



Wavelength, frequency and speed remain the same after diffraction. Spread due to diffraction

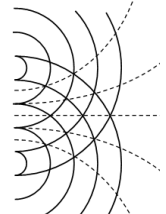
$$\propto \frac{\lambda}{w}, \text{ where } w \text{ is the width of the obstacle or opening.}$$

**Interference**



Two waves interfere when they cross or overlap each other. If they are in phase (crests meeting crests and troughs meeting trough), **constructive interference** is said to occur resulting in a wave with larger amplitude.

If they are half of a wavelength out of phase (crests meeting troughs), **destructive interference** occurs resulting in the destruction of both waves.

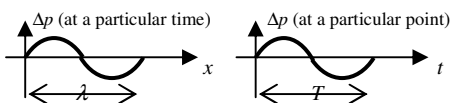


The diagram (left) is called an **interference pattern** of two circular waves generated by two sources producing the waves periodically and *in phase*. Wavelength, frequency and speed remain the same during and after interference.

**Constructive interference** occurs at places where a wave crest meets another crest, or a trough meets another trough. **Destructive interference** occurs at places where a crest and a trough meet.

The regions marked with dotted curves are called **antinodal lines** where constructive interference takes place. The regions between two adjacent antinodal lines are called **nodal lines** where destructive interference occurs.

Travelling **sound wave** through air is **longitudinal** because the air molecules oscillate parallel to the direction of propagation of the sound wave. A sequence of high (**compression**) and low (**rarefaction**) air pressure is generated and it propagates outward from the source carrying the sound energy with it.



$$f = \frac{1}{T}, \text{ speed of sound } v = \frac{\lambda}{T} = f\lambda$$

$$v(\text{solid}) > v(\text{water}) > v(\text{hotair}) > v(\text{coolair})$$

**v is constant** when sound travels in the same medium,

$$\therefore \lambda \propto \frac{1}{f} \text{ and } \frac{\lambda_2}{\lambda_1} = \frac{f_1}{f_2}$$

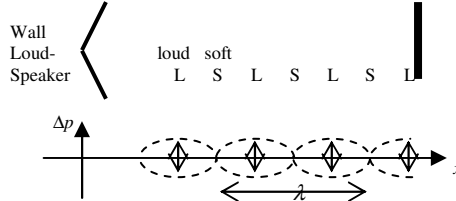
**f is constant** when sound travels from a medium into another,  $\therefore v \propto \lambda$  and  $\frac{v_2}{v_1} = \frac{\lambda_2}{\lambda_1}$

Sound has all the properties of a wave, namely reflection, refraction, diffraction and interference. An example of

refraction of sound is when it travels from a region into another region of different temperature. Sound travels faster in a warmer region (longer  $\lambda$ ) than in a cooler region (shorter  $\lambda$ ), and it bends towards the normal, i.e.  $\angle i > \angle r$ .

After **reflection**,  $f, \lambda$  and  $v$  remain the same.

When the forward and the reflected travelling waves superpose each other, a **standing wave** (a sequence of loud and soft sound at fixed positions quarter of a wavelength apart) is formed between the source and the wall. **Pressure antinodes** (max fluctuation in air pressure) give loud sound and **pressure nodes** (min fluctuation) give soft sound.



Every object has its own natural frequencies of vibration. If an energy source at one of these frequencies interacts with the object, the latter will be forced into vibration, a standing wave is formed. The object is in **resonance**. The natural frequencies of vibration are called **resonant frequencies**.

Standing waves in a **stretched string** of length  $L$ :

Overtones	Harmonics	$\lambda$	$f = v/\lambda$
Fundamental	first	$2L/1$	$1(v/2L)$
First	second	$2L/2$	$2(v/2L)$
Second	third	$2L/3$	$3(v/2L)$

Note:  $v$  is the speed of travelling wave in the string.

Standing waves in **open resonant tube** of length  $L$ : The vibration of the air column in the tube forms a standing wave. Has the same pattern of harmonics as strings but  $v$  is the speed of travelling sound wave in the tube.

Standing waves in **closed resonant tube** of length  $L$ :

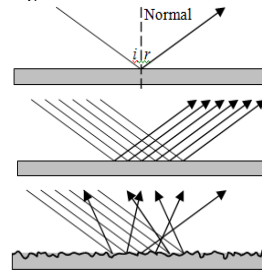
Overtones	Harmonics	$\lambda$	$f = v/\lambda$
Fundamental	first	$4L/1$	$1(v/4L)$
First	third	$4L/3$	$3(v/4L)$
Second	fifth	$4L/5$	$5(v/4L)$

For closed tubes only odd harmonics exist.

**The Doppler effect:** An approaching wave-emitting object 'compresses' the emitted wave in front of it resulting in shorter wavelength. The speed of the wave does not change

since it travels in the same medium,  $\therefore f = \frac{v}{\lambda}$  is higher. For a departing object, the wavelength of the emitted wave will be elongated and  $\therefore$  lower frequency.

**Light**

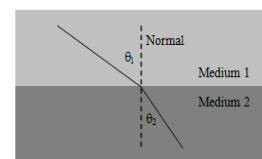


Reflection of a ray  
Law of reflection:  
 $\angle r = \angle i$

Specular reflection of a beam of parallel rays

Diffuse reflection of a beam of parallel rays

Refraction of a ray  
When a light ray enters a medium from a different medium at an angle to the normal, both direction and speed of the light ray change.



If the light ray travels from med 1 to med 2,  $\angle i = \theta_1$  and

$$\angle r = \theta_2, \text{ and the relative refractive index } r_{1to2} = \frac{\sin \theta_1}{\sin \theta_2}$$

The relative refractive index can also be determined from the **absolute refractive indices**  $n_1$  and  $n_2$ ,  $r_{1to2} = \frac{n_2}{n_1}$ .

If the light ray travels from medium 2 to medium 1, then  $\angle i = \theta_2$  and  $\angle r = \theta_1$ , and the **relative refractive index**

$$r_{2to1} = \frac{\sin \theta_2}{\sin \theta_1}. \text{ Also, } r_{2to1} = \frac{n_1}{n_2} = \frac{1}{r_{1to2}}$$

Hence  $n_1 \sin \theta_1 = n_2 \sin \theta_2$ , known as **Snell's law**.

When a light ray enters an optically denser medium (i.e. medium of higher refractive index), its speed decreases,

$$v \propto \frac{1}{n} \text{ and } \frac{v_2}{v_1} = \frac{n_1}{n_2}. \text{ In fact, } v = \frac{c}{n}, \text{ where } c \text{ is the speed of light in a vacuum } (c = 3.0 \times 10^8 \text{ ms}^{-1}).$$

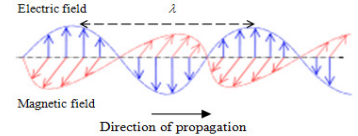
When a light ray enters an optically denser medium, it always bends towards the normal, hence  $\angle r < \angle i$ .

When it enters a **less dense** medium, it bends away from the normal, resulting in a phenomenon called **total internal reflection** if the angle of incidence  $\angle i$  is sufficiently large for the two media under consideration. The minimum angle of incidence for total internal reflection to occur is called the **critical angle**  $\theta_c$  for a particular colour (frequency) of light. A different critical angle results if a different combination of media is used.

Light reflects, refracts, diffracts and interferes with each other; therefore a **wave model** is a suitable tool to describe light phenomena.

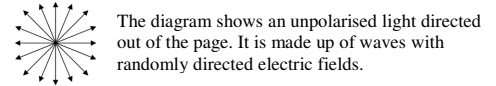
There is one important difference between light waves and the other waves mentioned previously. No medium is required for light to travel in. Light energy can be transferred from one position to another in a vacuum.

According to Maxwell light in space is an oscillating electric field associated with an oscillating magnetic field. These two fields are perpendicular to each other.



**Polarisation of light** Light emitted by an excited atom is polarised, i.e. the oscillating electric field lies on the same plane along the direction of propagation. Polarisation of light is a good indicator that light as a wave is transverse.

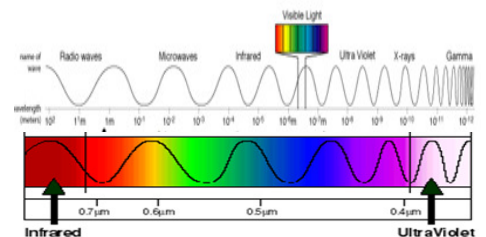
A light source (e.g. a fluorescent lamp) consists of many excited atoms emitting light independently. The light therefore consists of many independent waves that are randomly polarized about the direction of propagation. Such light is said to be **unpolarised**.



Each electric field can be resolved into two perpendicular components. Therefore unpolarised light can be considered as two perpendicular oscillating electric fields of the same amplitude.

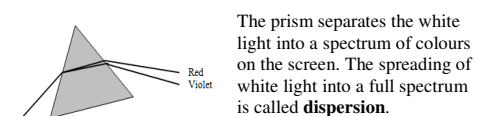
Unpolarised light can be made polarised by passing it through a polarising sheet (a Polaroid sheet). A polarising sheet has a particular polarising direction on its plane that allows those components parallel to this direction and removes components perpendicular to it. Theoretically the intensity of light is halved after passing through a polarising sheet. If a second sheet is placed in tandem with its polarising direction perpendicular to the first sheet, it is expected to block the rest from passing through.

**Electromagnetic spectrum:**



**Colour components of white light**

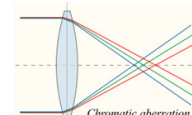
Light from the sun is called **white** light and found to compose of different colours (frequencies) of light. This can be demonstrated easily by passing a beam of sunlight through a triangular glass prism.



The prism separates the white light into a spectrum of colours on the screen. The spreading of white light into a full spectrum is called **dispersion**.

Dispersion occurs because the material of the prism refracts the different colour of lights to varying degrees. That is, the material has slightly different refractive indices for the different colours. Violet light (higher  $n$ ) is bent the most and red (lower  $n$ ) the least.

**Chromatic aberration**



In optics, **chromatic aberration** is a term used to describe the effect of a lens failing to focus all colours to the same point.

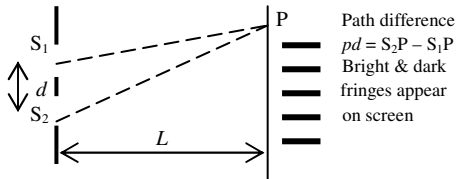
It occurs because lenses have a different refractive index for different frequencies of light. The refractive index increases with increasing frequency. Chromatic aberration appears as fringes of colour along boundaries that separate dark and bright parts of the image.

**Speed of light in a medium** All light, irrespective of its colour, travels at the speed of light  $c$  in a vacuum, i.e.

$3.00 \times 10^8 \text{ ms}^{-1}$ . Once it enters a medium its speed depends

on the refractive index of the medium for its colour,  $v = \frac{c}{n}$ .

Thermal oscillations of electrons in atoms give off electromagnetic radiation (visible light). In an incandescent light bulb, the atoms in the filament are excited by heating, and they give off their excess energy as wave trains (with **wide spectrum** of wavelengths) of light, each lasts about  $10^{-8}$ s. The emitted light is the sum of such wave trains that bear a **random phase** relation to each other and they are **incoherent**. Two light globes produce incoherent light, hence no **interference pattern**. Thomas Young demonstrated the **wave nature** of light with his **double-slit experiment** to obtain an interference pattern. He used sunlight through a narrow slit as the light source and then through the double slits. Lights through the double slits are **coherent** because they are split from the same wave trains from the single slit,  $\therefore$  there is an interference pattern. **Laser** is a very coherent source because it is **monochromatic** (single wavelength) and wave trains are emitted simultaneously, hence a very clear interference pattern.

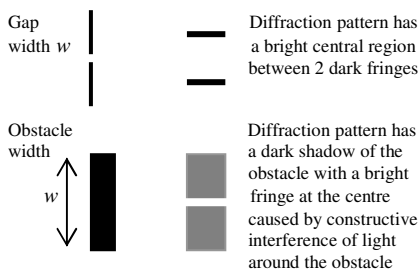


Constructive interference (bright):  $pd = 0\lambda, 1\lambda, 2\lambda, \dots$

Destructive interference (dark):  $pd = 0.5\lambda, 1.5\lambda, 2.5\lambda, \dots$

**Spacing between fringes** increases when wavelength  $\lambda$  increases, screen distance  $L$  increases and/or slits separation  $d$  decreases.  $\lambda_{red} > \lambda_{green} > \lambda_{blue} > \lambda_{violet}$ .  $f = \frac{c}{\lambda}$ .

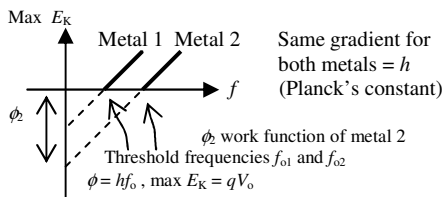
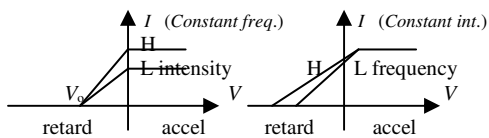
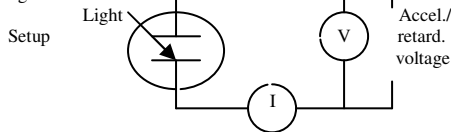
**Diffraction** of light also demonstrates the wave nature of light.



Extent of diffraction  $\propto \frac{\lambda}{w}$ . Significant effect when  $\frac{\lambda}{w} \approx 1$ .

More diffraction when  $\lambda$  is longer and/or  $w$  is smaller.

**Photoelectric effect** demonstrates particle-like nature of light.



Failure of the wave model to explain the photoelectric effect According to the wave model, light is a continuous wave and the intensity is related to its amplitude, which measures the energy of the wave. Therefore an electron can absorb any amount of light energy, depending on the time interval it is exposed to the light wave.

The wave model failed to explain why

- (1) maximum kinetic energy remained the same when the intensity was changed
- (2) maximum kinetic energy changed with the frequency of light used
- (3) there was a threshold frequency for each metal used.

Einstein's interpretation of the photoelectric effect-the photon model: A beam of light is a stream of particles called photons. Light of a single frequency  $f$  consists of photons of the same energy  $E = hf = hc/\lambda$ .

There are more photons in a more intense beam, hence higher current. When photons strike a metal, some will be absorbed by the electrons in the metal. To have photoelectrons emitted, the energy of each photon must be high enough for the electrons to overcome the bonding energy (i.e. the work function  $\phi$ ). As the photons penetrate into the metal they collide with other electrons before they are absorbed. Each collision lowers the photon frequency (energy) slightly, the **Compton effect**.  $\therefore$  electrons at the surface escape with higher (max) kinetic energy than those inside metal,  $\max E_K = hf - \phi$  for surface electrons. The emitted electrons have a range of kinetic energy. When the retarding voltage increases, more electrons will be stopped and the current decreases.

The Compton effect and photon momentum: The particle nature of light was further supported by the Compton effect.

Photon momentum  $p = \frac{E}{c} = \frac{h}{\lambda}$ .

The two models (wave and particle) of light appear to be inconsistent with each other but both have been shown to be valid depending on the circumstances. This dual nature of light is known as wave-particle duality.

Wave nature of matter: de Broglie proposed that a moving material particle also has wave-particle duality. Wavelength of particle is related to its momentum (like a photon).

de Broglie  $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE_K}}$ . These equations are

valid when  $\lambda$  in m,  $m$  in kg,  $v$  in  $\text{ms}^{-1}$ ,  $E_K$  in J,  $h$  in Js.

The diffraction of electrons from the surface of a metal crystal confirmed the wave nature of matter.

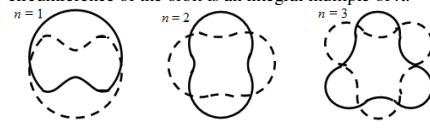
An electron with the same  $\lambda$  as a photon has the same momentum as the photon,  $p = h/\lambda$ .

When a gas or metal vapour is heated, the gas or vapour glows and emits a characteristic diffraction pattern (obtained with a **diffraction grating**) called an **emission spectrum**. When sunlight passes through a gas/vapour, some dark lines appear in its spectrum called **absorption spectrum**, caused by the absorption of certain wavelengths of sunlight by the atoms or molecules in the gas/vapour. These spectra are evidence for **quantised atomic energy levels**.

Ionisation	-----	$E = 0 \text{ eV}$
	$n = 4$	$-0.85 \text{ eV}$
Second excited state	$n = 3$	$-1.5 \text{ eV}$
First excited state	$n = 2$	$-3.4 \text{ eV}$
Ground state	$n = 1$	$-13.6 \text{ eV}$

Electrons move around a nucleus with **discrete** energies. When an electron jumps from high to low energy level, it loses energy in discrete amount equal to the difference between the two levels and results in emission of a photon of the same energy.  $hf = E_H - E_L$ .

De Broglie used the idea of **standing matter waves** to explain the quantised energy levels of an atom. The only matter waves that persist are those for which the circumference of the orbit is an integral multiple of  $\lambda$ .



**Production of light from matter**

**Incandescent lights:** See the first column on this page.

**LEDs:** When a small potential difference is applied to a semi-conducting device, LED, electrons jump from an energy level (valence band) to a higher energy level (conduction band). When they drop back into the valence band, they release energy as photons. The colour of light emitted depends on the energy difference between the two energy levels (valence and conduction bands).

**Lasers:** In lasers light is produced from gas atoms. Most of the gas atoms in a laser are in an excited state. If a photon of appropriate energy is introduced to the gas, it stimulates the gas atoms to release energy as photons systematically. Laser light is usually polarised, coherent (in phase) and has a very narrow band of wavelength.

**Synchrotron light:** Electrons are accelerated to close to the speed of light by magnets in a synchrotron. During acceleration the electrons produce electromagnetic radiation known as synchrotron light. The light are extremely bright, highly polarised, emitted in very short pulses and has a broad range of wavelengths.