



**Online & home tutors** Registered business name: itute ABN: 96 297 924 083

***2019***  
***Further***  
***Mathematics***

***Year 12***  
***Problem Solving Task***  
***Module: Geometry and***  
***measurement***

***Time allowed: 2 hours plus***

## Theme: Garden Geometry

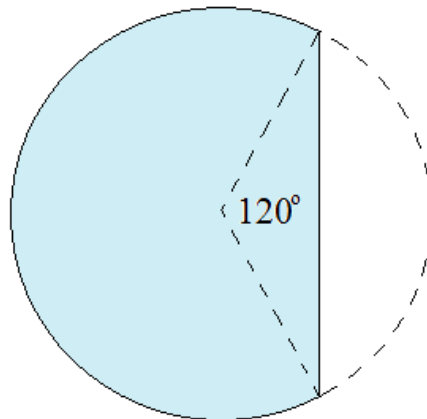
**The task:** A garden is under construction.

Questions/problems require answers/solutions. Your task is to help to answer/solve them. Reasoning, working and/or solution process must be shown fully.

Unless stated otherwise, you may write your answers corrected to 4 decimal places, or in exact form. Angles are measured in degrees.

### Problem I Goldfish pond

A pond of constant depth in the shape of a segment of a circle at the water surface is to be built. The angle subtended by the chord at the centre of the circle is  $120^\circ$ . The length of the chord is 5.0 m. The depth of the pond is 0.50 m.



- Calculate the radius of the circle.
- Calculate the area of the water surface.
- Calculate the volume of water required to fill the pond to a depth of 0.5 m.

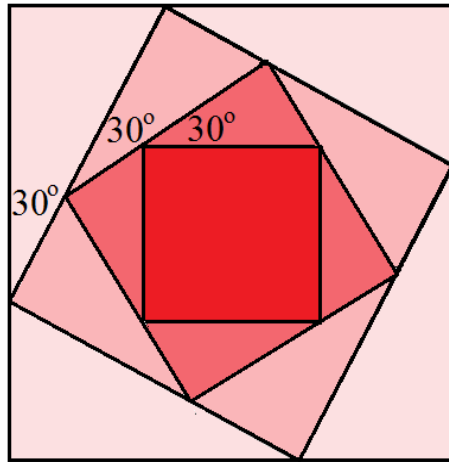
**d.** Calculate the perimeter of the pond.

**e.** Calculate the radius of the circle if the angle subtended at the centre of the circle is reduced to  $60^\circ$ , whilst the depth and volume of water required remain the same.

**f.** Calculate the angle to the nearest degree subtended by the chord at the centre of the circle if  $\frac{\text{area of minor segment}}{\text{area of major segment}} = \frac{1}{4}$ .

## Problem II Flower beds

Four flower beds are cultivated. The beds are bounded by four squares. The squares have a common centre and rotated by  $30^\circ$  relative to each other as shown in the diagram.



- a.** A corner of a smaller square divides the side of the larger square into a ratio of  $1:m$  where 1 and  $m$  correspond to the shorter and longer lengths respectively. Determine the value of  $m$ .
- b.** Hence draw accurately on the diagram above a fifth square flower bed with corners touching the innermost square and rotated  $30^\circ$  clockwise relative to it.
- c.** Determine the length scale factor for a pair of touching squares.
- d.** Is the length scale factor constant for all pairs of touching squares? Justify your answer by showing that for another pair.

**e.** Determine the area scale factor for a pair of touching triangles.

Is the area scale factor constant for all pairs of touching triangles? Justify your answer by showing that for another pair.

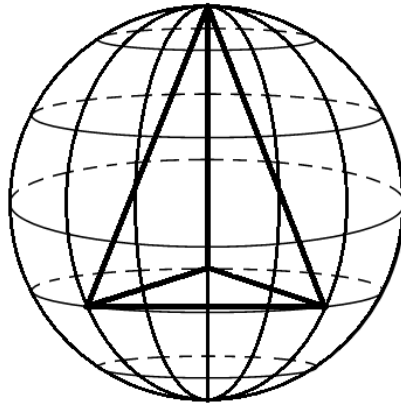
**f.** If the area of the region enclosed by the smallest square (the fifth one drawn by you) is a metre square. Calculate the area of the region enclosed by the largest square.

**g.** If the side length of the largest square is 10 metre. Calculate the side length of the smallest square.

### Problem III Garden sculpture

A garden sculpture consists of a wire-frame sphere (representing Earth) with a thicker-wire-frame tetrahedron (triangular pyramid) inside it.

The wire frame of the sphere imitates the set of lines of longitude and latitude of Earth. The lines of longitude are  $30^\circ$  apart on the sphere, and the lines of latitude are also  $30^\circ$  apart. The radius of the sphere is 2.0 m.



- a. Calculate the length of a sloping side and the length of a horizontal side to show the tetrahedron is **not** regular.
- b. Calculate the height of the tetrahedron.
- c. Calculate the *shortest* distance of travel along the surface of the sphere from one vertex of the tetrahedron back to the same vertex, visiting the other three vertices on the way.

**d.** The tetrahedron is a wire-frame structure and has no walls. Calculate the radius of the largest sphere that fits inside the tetrahedron.

**e.** Calculate the total surface area of the tetrahedron if it is a solid.

**f.** If the sphere is a solid,  
calculate the value of the ratio  $\frac{\text{volume inside the tetrahedron}}{\text{remaining volume of the sphere outside the tetrahedron}}$ .

### Problem IV Patterned garden brick wall

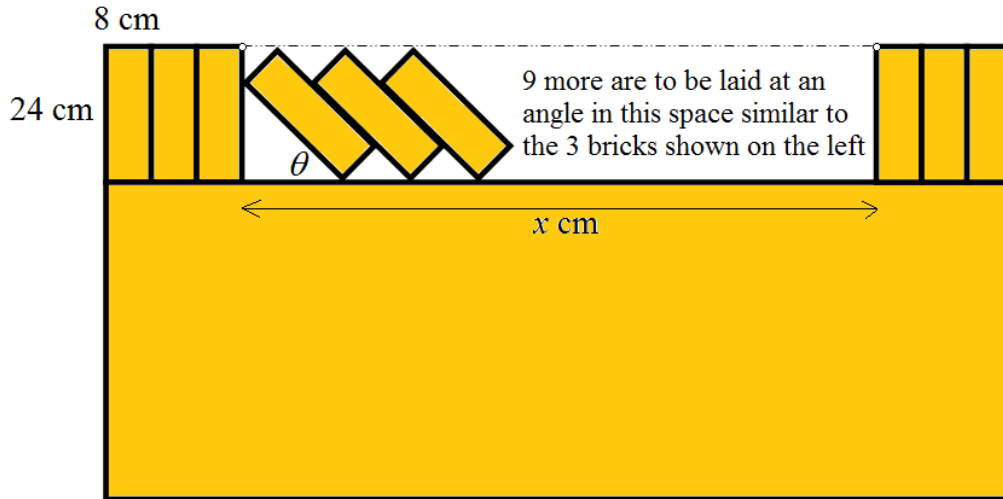
The brick wall is to be built with the same rectangular bricks.

The measurements (including mortar) of the exposed surface of a brick are 24 cm by 8 cm.

A section of the garden wall is shown below.

12 sloping bricks are to be laid as shown and fit in the space marked as  $x$  cm in the diagram.

Note that the top of each sloping brick levels with the top of a vertical brick.



a. Determine angle  $\theta$  between a sloping brick and the horizontal.

b. Calculate the minimum value of  $x$ .

**End of Problem Solving Task**