2019 Mathematical Methods

Year 12 Modelling Task

Time allowed: 2 hours plus

Modelling Task

Theme: Sound waves

Sound from a source (e.g. a loudspeaker) causes continuous variation in air pressure in front of the source. This continuous variation in air pressure forms a sound wave in space. The amplitude of a sound wave is the maximum variation in air pressure.

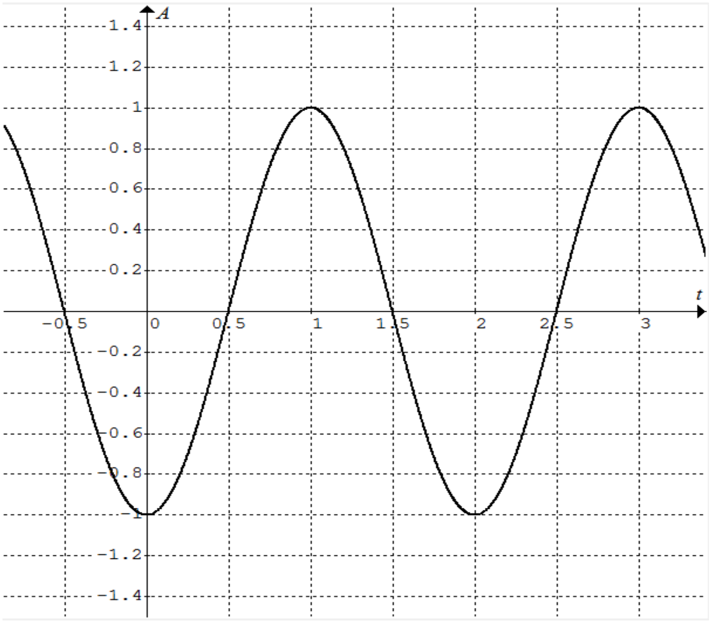
Assumed knowledge:

Circular functions, logarithmic and exponential functions, power functions, sum of functions, addition of ordinates, transformations, calculus and CAS

Note: Arbitrary units are used for amplitude, time and distance.

Part I (65 – 75 minutes)

Q1

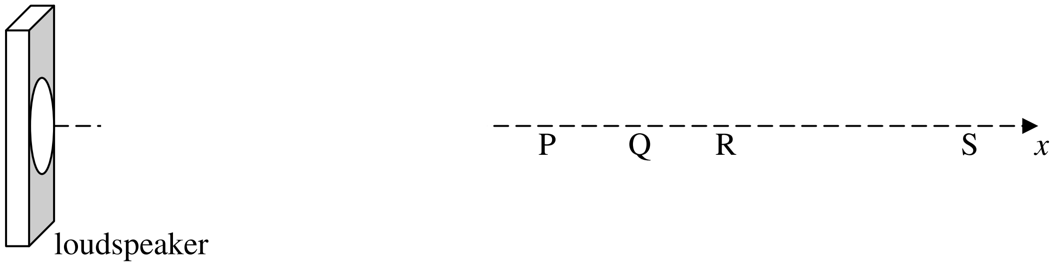


a. State the amplitude and period of the graph above.

b. Write the equation for the graph above in the form $y = h\cos(at+b)$ where $a, b \in R$ and $h \in R^{+}$.

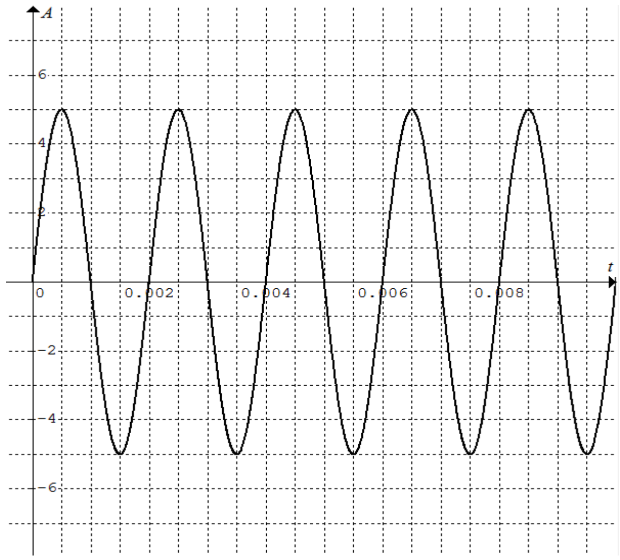
c. Write the equation for the graph above in the form $y = k\sin(ct+d)$ where $c, d \in R$ and $k \in R^{+}$.

Sound from a loudspeaker is measured at four different locations P, Q, R and S directly in front of it.

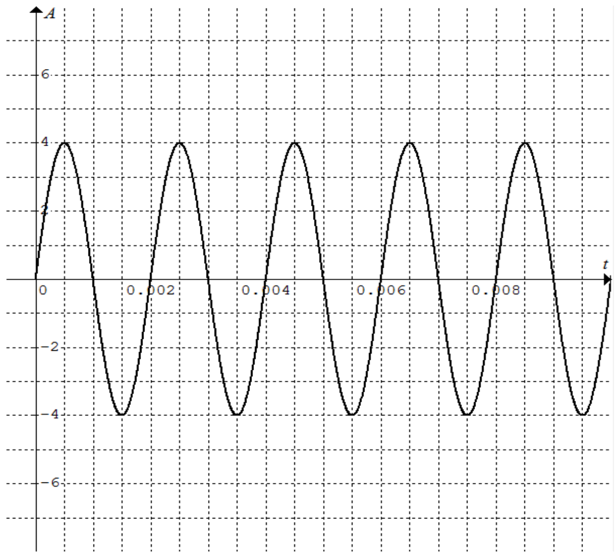


The measurements of the sound wave at locations P, Q, R and S are displayed below.
x is the distance of a location from the loudspeaker.

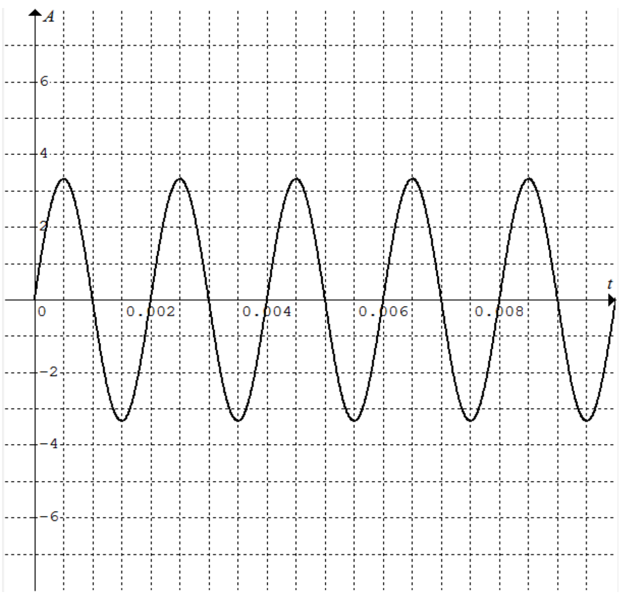
P $x = 4$



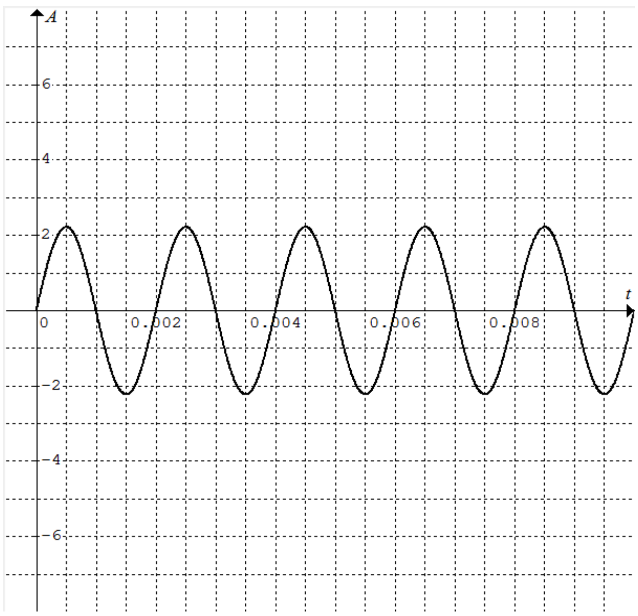
Q $x = 5$



R $x = 6$



S $x = 9$



Q2

a. Use the sine function to model the sound wave at location P.

b. Discuss the changes in the measurements when the sound detector (a microphone) moves from P to S.

c. Complete the following table.

Location	Distance from loudspeaker x	Amplitude A
P		
Q		
R		
S		

d. Plot a graph of A versus x.

e. A possible model to describe the relationship between x and A is an exponential function of the form $A = a \times e^{bx}$ where $a, b \in R$.

(i) Use the following steps to determine the values of a and b to fit the data in the table

Step 1: Plot the graph of $\log_e A$ versus x

Step 2: Draw the line of best fit

Step 3: b is the gradient of the line of best fit, and $\log_e a$ is the vertical axis intercept

(ii) Give an explanation of Step 3 in finding a and b.

f. Another possible model is a power function in the form $A = k \times x^n$ where $k, n \in R$.

(i) Determine the values of k and n to fit the data in the table using the following steps.

Step 1: Plot the graph of $\log_e A$ versus $\log_e x$

Step 2: Draw the line of best fit

Step 3: n is the gradient of the line of best fit, and $\log_e k$ is the vertical axis intercept

(ii) Give an explanation of Step 3 in finding k and n.

g. Decide and explain which one is a better model.

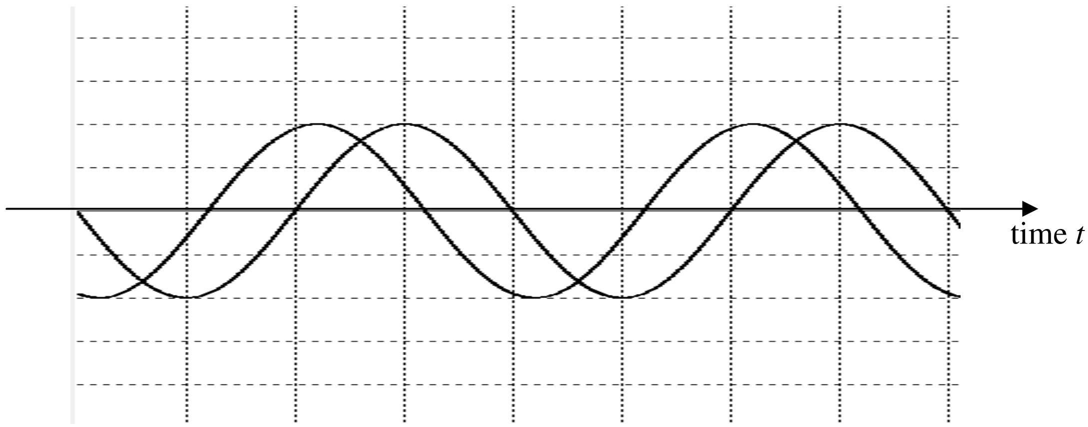
h. Use the better model to find the rate of change of amplitude A with respect to distance x from the loudspeaker.

Part II (55 – 65 minutes)

The single loudspeaker in Part I is now replaced by two loudspeakers placed side-by-side.
Each loudspeaker sends out a sound wave.
Investigate the combined effects of the two sound waves at certain location directly in front of the two side-by-side loudspeakers by adding the two waves.

Case 1 The two waves have the same period and amplitude, but they are out of phase, i.e. one wave is emitted earlier than the other.

a. Use the method of addition of ordinates to find the resulting wave of the two out of phase waves in the time interval as shown below.



b. Compare the amplitude and period of the resulting wave with the individual waves.

c. Write down the equations of your own two waves having the same period and amplitude but are out of phase by a quarter of a period.
Then use CAS to sketch the graph of the **sum** of your own two waves over several periods.
If you do not know the equations, then use $A = \sin\left(\pi t + \frac{\pi}{4}\right)$ and $A = \sin\left(\pi t - \frac{\pi}{4}\right)$.

Bonus marks for using your own equations.

d. Determine the equation of the resulting wave in part c.

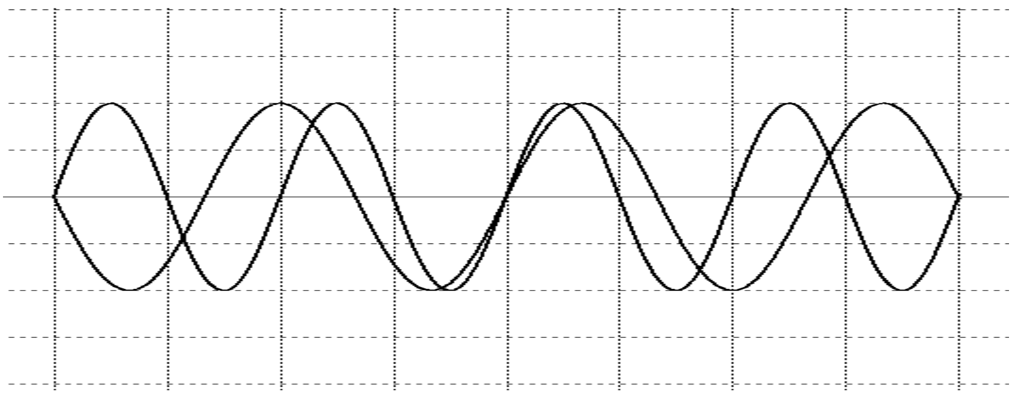
Case 2 Consider two waves from the loudspeakers at certain location directly in front of the loudspeakers. They have equations $A=\frac{1}{2}\sin(2\pi t)$ and $A=\sin\left(2\pi\left(t-\frac{1}{2}\right)\right)$.

e. Compare the amplitudes, periods and phase of the two waves.

f. Use CAS to sketch the graph of the sum of the two waves.
Compare the amplitudes and periods of the resulting wave with the individual waves.

g. Determine the equation of the resulting wave in part f.

Case 3 The two waves have the same amplitude but different periods.
The following diagram shows two such waves in a time interval.



a. On the diagram above, use the method of addition of ordinates to sketch the resulting wave of the sum of the two waves.

Now consider two waves given by $A = \sin(2\pi t)$ and $A = \sin\left(\frac{2\pi}{1.1}t\right)$.

b. Find the difference in the period of the two waves.

c. Use your CAS to investigate (no sketching required) the resulting wave of the sum of the two waves for $t \in [0, 50]$. Describe the important features of the resulting wave.

d. Find the period T_s of the shape (similar to the one in part a) in the resulting wave.

e. Find the relationship between the period T_A of $A = \sin\left(\frac{2\pi}{1.1}t\right)$ and the period of the shape in part d.

End of Task

