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2019 Specialist Mathematics

Year 12 Problem Solving Task

(Time allowed: 2.0 hours plus)

Problem Solving Task

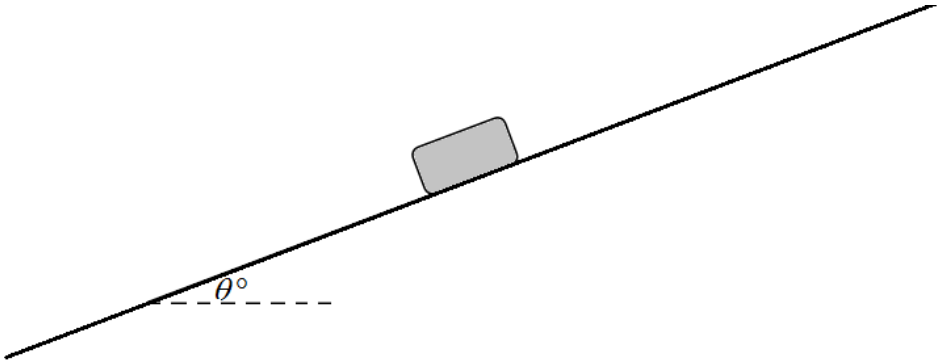
Theme: Up and down a slope

Assumed knowledge:
Dynamics, force and components, Newton's laws, friction and air resistance, kinematics, calculus including differential equations, use of CAS

In this task object is referred to as particle. Take g *as* $9.8\,N\,kg^{-1}$.
Force is measured in newtons (N), distance in metres (m), time in seconds (s) and mass in kilogram (kg).
As a mathematical exercise, assume that air resistance has the same form and friction force remains constant throughout the motion of a particle.

Part I (60 – 75 minutes)

A particle is projected up a plane inclined at $\theta°$ to the horizontal.
Its motion is affected by force of gravity F_g, friction force F_f and air resistance force F_a.
Let the mass of the particle be 1.0 kg. Assume $F_a = 0.010 \times v^2$ and $\theta = 30$.



a. The initial ($t = 0$) speed of the particle is 10.0 m s^{-1} at $x = 0$.
On the diagram above draw labelled arrows of proportional length and appropriate direction to represent the forces on the particle, assuming the friction force $F_f = 0.2 \times F_g$.

b. Take the direction of motion as the positive *x* direction.
Show that the resultant force on it is approximately $0.010\left(660 + v^2\right)$ down the slope while the particle is on its way up the slope.

c. Use the equation of motion to write an appropriate differential equation involving v and x.

d. Show that $x \approx 50\log_e\left(\frac{760}{660+v^2}\right)$ by solving the differential equation in part c without using CAS.

e. Show that the maximum distance travelled by the particle up the slope is 7.054 m before reversing its direction of motion.

f. Determine the time required to travel the maximum distance.

g. The particle has reversed its direction. Again take the direction of motion (down the slope) as the positive x direction, and assume $F_a = 0.010 \times v^2$.

Show that the resultant force on it is approximately $0.010\left(320 - v^2\right)$ down the slope while the particle is on its way down the slope.

h. Write a differential equation involving v and x.

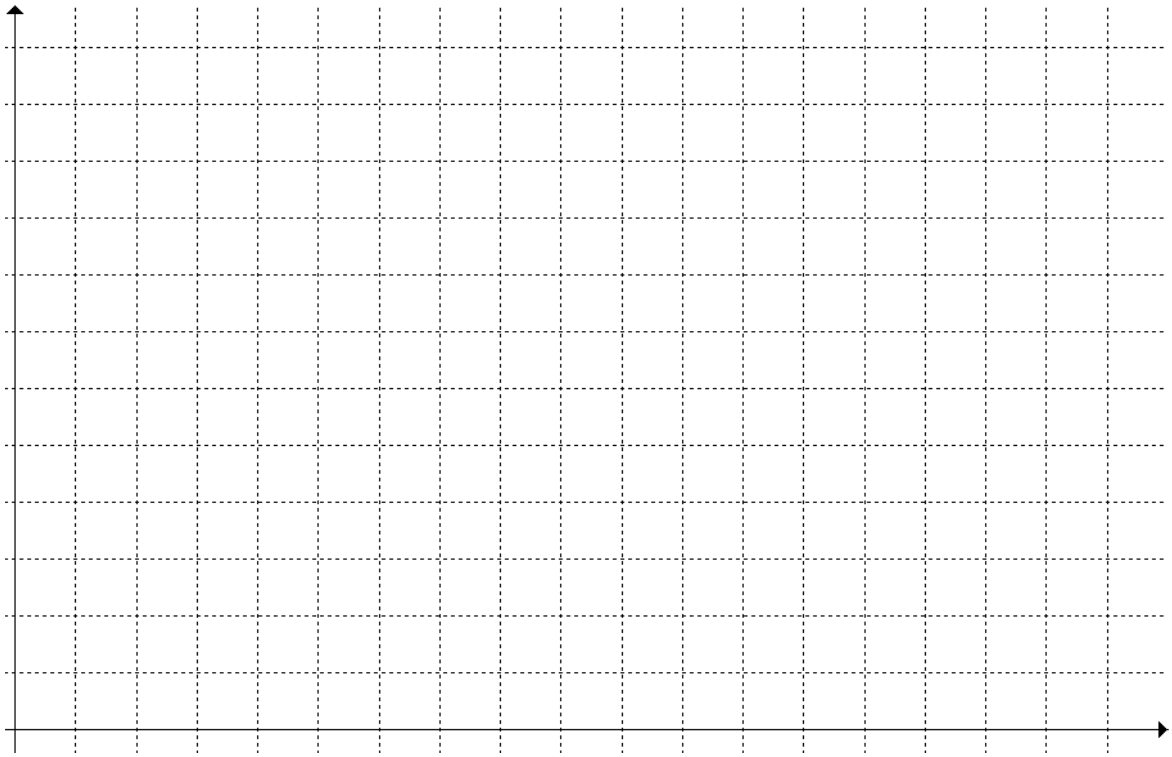
Solve it without using CAS to show that $x \approx 50 \log_e\left(\frac{320}{320 - v^2}\right)$.

i. Calculate the distance travelled by the particle when its speed is back to 10.0 m s^{-1}.

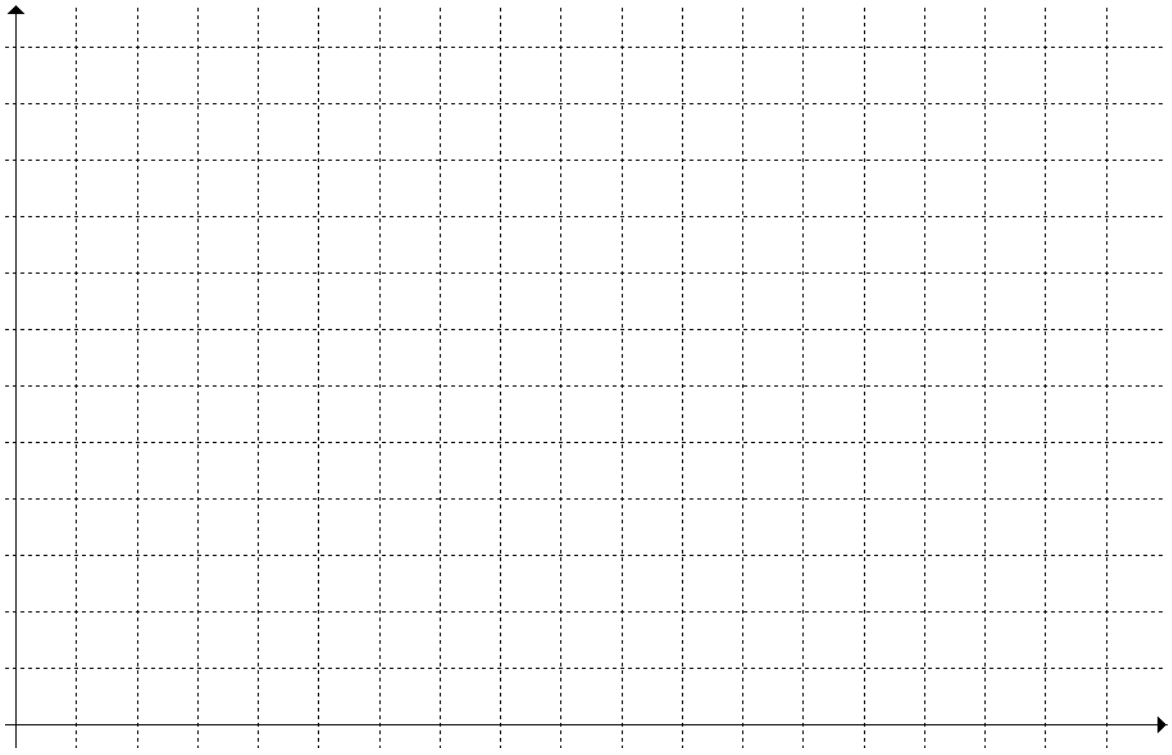
j. Determine the ultimate speed of the particle if the slope has no end.

k. Is it possible to keep the particle at the highest point without reversing its motion down the slope by changing the angle between the slope and the horizontal? Explain.

l. Sketch the speed v versus position x graph for the motion up the slope.



m. Sketch the speed v versus position x graph for the motion down the slope.



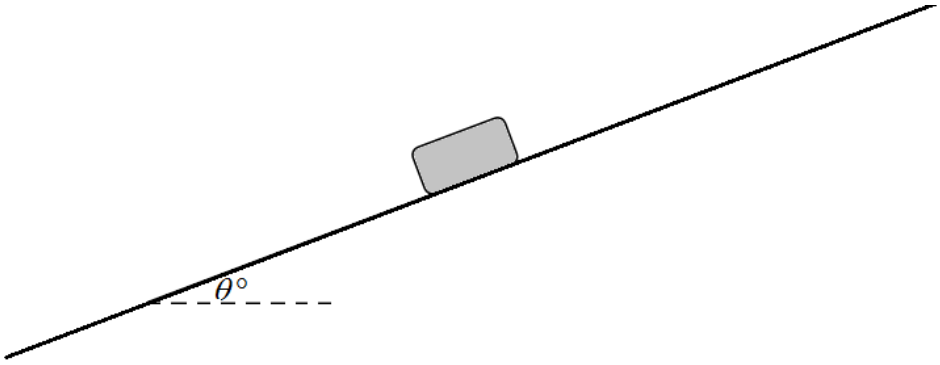
End of Part I

Part II (60 – 75 minutes)

In this task object is referred to as particle. Take g *as* $9.8\,N\,kg^{-1}$.
Force is measured in newtons (N), distance in metres (m), time in seconds (s) and mass in kilogram (kg).

As a mathematical exercise, assume that air resistance has the same form and friction force remains constant throughout the motion of a particle.

Another particle of a different shape is projected up a plane inclined at $\theta° = 30°$ to the horizontal.
Its motion is affected by force of gravity F_g, friction force F_f and air resistance force F_a.
Let the mass of the particle be 1.0 kg.
Assume $F_a = k\,v$ where $k \in R^+$, and friction force $F_f = 0.2\cos\theta° \times F_g$.



The initial ($t = 0$) speed of the particle is 10.0 m s^{-1} at $x = 0$.

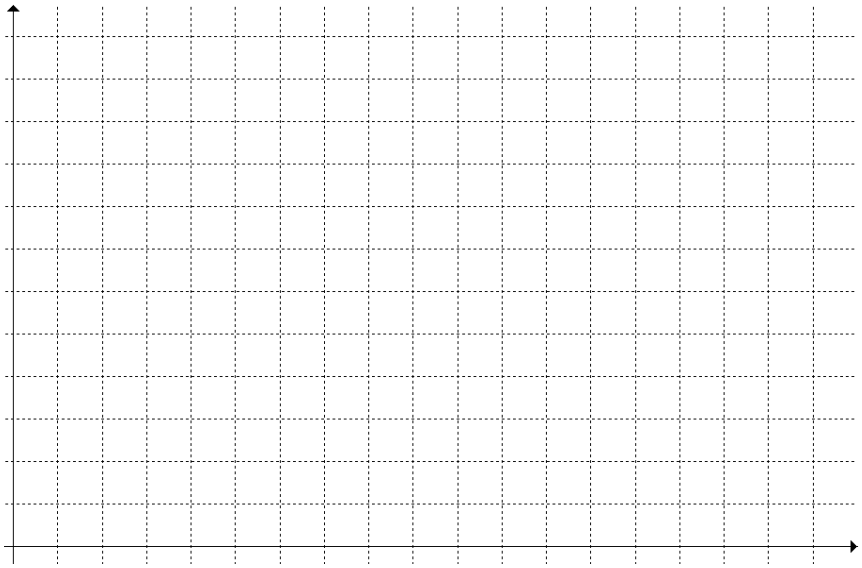
a. Take the direction of motion as the positive x direction.
Show that the acceleration (m s^{-2}) of the particle is approximately $6.5974 + kv$ down the slope while the particle is on its way up the slope.

b. Write an appropriate differential equation and without using CAS solve it to show
$x \approx \frac{1}{k}(10 - v) + \frac{6.5974}{k^2}\log_e\left(\frac{6.5974 + kv}{6.5974 + 10k}\right)$ where $k \in R^+$.

c. If the maximum distance travelled by the particle up the slope is also 7.054 m, same as the first particle in Part I, show that $k \approx 0.0743$.

d. Find the time taken by the second particle to travel the maximum distance 7.054 m up the slope.

e. Graphically investigate the effects of changing the value of k on the maximum distance travelled by the second particle up the slope for $k \in R^+$.
Discuss your findings in terms of the rate of change in the maximum distance with respect to k.



f. Discuss the situation when $k = 0$ and find the maximum distance travelled by the second particle up the slope.

g. Find the time taken by the second particle to travel the maximum distance up the slope in part f.

h. The second particle has reversed its direction. Again take the direction of motion (down the slope) as the positive x direction, and assume $F_a = k v$, $k \in R^+$, and $x = 0$ initially.
Write a differential equation involving v and t.
Solve it without using CAS to find $v(t)$, i.e. v as a function of t with k as a parameter.

i. Hence without using CAS find $x(t)$ with k as a parameter.

j. Hence find $v(x)$ with k as a parameter.

k. Find the distance covered by the second particle when it is travelling at a half of its ultimate speed.

End of Part II