2019 VCAA Further Mathematics Exam 2 Solutions

SECTION A - Core
Data analysis

Q1a Day number

Q1b

4	1	8
5		
6	0	7
7	0	5 7
8	0	6
9	0	2 8
10	7	
11	8	
12	7	

Q1ci $Q_1 = 12.2$

Q1cii $\frac{3}{15} = 0.20 = 20\%$

Q2ai $IQR = 17 - 12 = 5°C$

Q2aii $25 - 15 = 10°C$

Q2aiii 1

Q2b $50\% \times 30 = 15$ days

Q3 The *median minimum daily temperature* for the month (the response variable) varies with the *month* (the explanatory variable). The median decreases with the month.

Q4a *humidity 9 am*

Q4b *humidity 3 pm* $= -1.26 + 0.765 \times$ *humidity 9 am*

Q4c By CAS, $r \approx 0.871$

Q5a *pressure 3 pm* increased by 0.8894 hPa for 1 hPa increase in *pressure 9 am*

Q5b *pressure 3 pm* $= 111.4 + 0.8894 \times 1025 \approx 1023$

Q5c Interpolation

Q5d Prediction $= 111.4 + 0.8894 \times 1013 \approx 1012.36$
Residual $\approx 1015 - 1012.36 \approx 3$ hPa

Q5ei $r = b\frac{s_x}{s_y} \approx 0.8894 \times \frac{4.5477}{4.1884} \approx 0.9657 \approx 0.966$

Q5eii $r^2 \approx 0.966^2 \approx 0.933 = 93.3\%$

Q5fi Assumption: There is a linear relationship between the atmospheric pressure at 3 pm and the atmospheric pressure at 9 am.

Q5fii The residual plot is not random enough to support the assumption.

Q6a

	Summer	Autumn	Winter	Spring
SI	*0.89*	*1.00*	1.41	*0.70*

Q6b $\frac{262}{1.41} \approx 186$ mm

Recursion and financial modelling

Q7a $V_0 = 60000$, $V_1 = 0.9 \times 60000$, $V_2 = 0.9^2 \times 60000 = 48600$

Q7b $0.1 = 10\%$

Q7c $0.9^n \times 60000 < 20000$, $n > 10.43$.: after 10 years, i.e. sometime in the 11^{th} year

Q7d $V_{n+1} = V_n - 0.08V_0$ where $V_0 = 60000$

Q8a \$3700

Q8b $0.0035 \times 12 = 0.042 = 4.2\%$

Q8c By Finance Solver, find N = 60.02495091 for FV = 0, set N = 60 to find FV ≈ 92.15. After 60 months the balance is \$92.15

Q8d $200000 \times \frac{4.2}{100 \times 12} = 700$ dollars

Q9a By Finance Solver, set FV = −262332.33 to find PMT ≈ −1704.03. Each fortnightly repayment is \$1704.03

Q9b $1704.03 \times 78 - (350000 - 262332.33) \approx 45246.67$ dollars

Q9c $1 + \frac{4.8}{12 \times 100} = 1.004$
$B_0 = 262332.33$, $B_{n+1} = 1.004B_n - 3517.28$

SECTION B - Modules

Module 1: Matrices

Q1a Order 3×2

Q1b $50+20+40=110$

Q1c

$$L=\begin{bmatrix}50\\20\\40\end{bmatrix}$$

Q1d

$$R^T=\begin{bmatrix}3 & 6 & 22 & 19\\1 & 10 & 7 & 2\\1 & 3 & 10 & 26\end{bmatrix}$$

Q1e e_{32} represents the number of cars parked for 2 hours in area *C*.

Q2a $\dfrac{400}{2000}=0.2=20\%$

Q2b

$$S_1=\begin{bmatrix}\mathbf{300}\\\mathbf{780}\\300\\\mathbf{620}\end{bmatrix}\begin{matrix}A\\F\\G\\W\end{matrix}$$

Q2c $0.1\times 600+0.2\times 600=180$, $\dfrac{180}{300}=0.6=60\%$

Q2d

$$M=\begin{bmatrix}0 & 1 & 0 & 0\\1 & 0 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

Q3a Location *A*, $0.3\times 500+0.4\times 600+0.6\times 500+0.3\times 400=810$

Q3b

$$B_1=\begin{bmatrix}-210\\0\\210\\0\end{bmatrix}$$

Q3c

$$R_1=V\times R_0+B_1=\begin{bmatrix}810\\300\\310\\580\end{bmatrix}+\begin{bmatrix}-210\\0\\210\\0\end{bmatrix}=\begin{bmatrix}600\\300\\520\\580\end{bmatrix}$$

$$VR_1=\begin{bmatrix}786\\288\\282\\644\end{bmatrix},\ R_2=\begin{bmatrix}600\\288\\512\\600\end{bmatrix}$$

Module 2: Networks and decision mathematics

Q1a Office

Q1bi Hamiltonian cycle

Q1bii Office --- science laboratories --- mathematics class rooms --- gymnasium --- computer rooms --- library --- office

Q2a

Student	Sport
Blake	Tennis
Charli	Football
Huan	Basketball
Marco	Athletics

Q2a

Student	Sprinting distance (m)
Anita	400
Imani	200
Jordan	100
Lola	300

Q3a 8

Q3b Start of week 13

Q3c *J*

Q3d $2+4+7+1+2+2+5+6=29$ weeks

Q3e

Activity	Reduction (weeks)
C	0
D	1
G	2
H	1
K	1

Overall completion time to be reduced by 4 weeks at a minimum cost of $2000\times 1+2500\times 2+1000\times 1+4000\times 1=12000$ dollars (this answer is not required)

Module 3: Geometry and measurement

Q1a Area of the shaded region $= 160\times 40 + 12\times 25 = 6700$ m^2

Q1b Volume $= 6\times 2.4\times 2.6 = 37.44$ m^3

Q1c Total surface area $= 2\left(6\times 2.4 + 6\times 2.6 + 2.4\times 2.6\right) = 72.48$ m^2

Q1d $8\times 3\times 2 = 48$

Q2a Sydney (34° S) is closer to the equator than Magadan (60° N).

Q2b Distance $= 6400\times \dfrac{\pi}{180}\times (34+60) \approx 10500$ km

Q2c 6 am Sydney time = 4 am Perth time
Number of hours between 4 am 1 June and 10 am 11 June
$= 24\times 10 + 6 = 246$

Q3ai Distance $AC = \sqrt{25^2 - 15^2} = 20$ m

Q3aii $\angle ACB = \sin^{-1}\left(\dfrac{15}{25}\right) \approx 37°$

Q3b The cosine rule:
Distance $RP = \sqrt{20^2 + 38^2 - 2(20)(38)\cos 120°} \approx 51$ m

Q3c Vertical distance from M to the container
$= \sqrt{4.4^2 - \left(\dfrac{2.4}{2}\right)^2 - \left(\dfrac{6}{2}\right)^2} \approx 3$ m

Module 4: Graphs and relations

Q1a 60

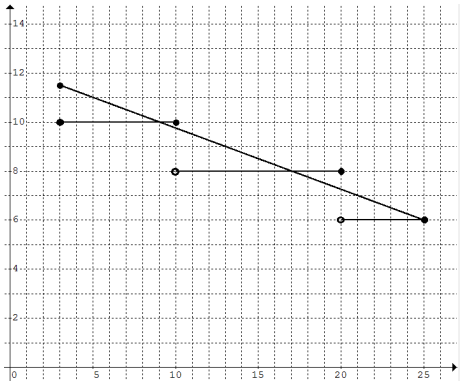
Q1bi Average rate $= \dfrac{32-62}{5} = -6$

Q1bii $32 - 6\times 3 = 14$

Q2a 3

Q2b ≤ 10

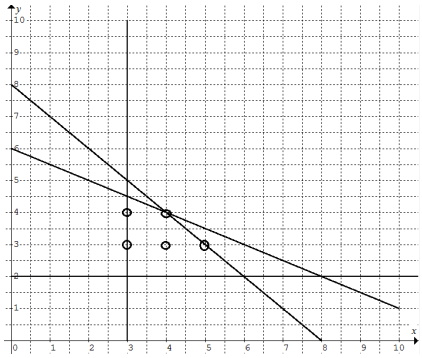
Q2c



Q2d 9, 17, 25

Q3a $5x + 10y \leq 60$

Q3b



Q3c 4 cars and 4 minibuses, cost $= 70\times 4 + 100\times 4 = 680$ dollars

Q3d 3 cars and 4 minibuses, cost $= 70\times 3 + 100\times 4 = 610$ dollars

Q3e Objective function: Cost $C = mx + 100y$
Point $(6,2)$ gives maximum cost when $m > 100$
$\therefore\ 70 + k = 100,\ k = 30$

Please inform mathline@itute.com re conceptual and/or mathematical errors