2019 VCAA Mathematical Methods Exam 1 Solutions

Q1ai $f'(x)=\frac{-3}{(3x-1)^2}$, $x>\frac{1}{3}$

Q1aii $\int f(x)dx=\frac{1}{3}\log_e(3x-1)+c$, an antiderivative of $f(x)$ is $\frac{1}{3}\log_e(3x-1)$

Q1b $g'(x)=\frac{(x+1)\pi\cos\pi x-\pi\sin\pi x}{(x+1)^2}$, $g'(1)=\frac{-\pi}{2}$

Q2a Eq. of f^{-1} is $x=\frac{1}{3y-1}$, .: $y=\frac{1}{3}\left(\frac{1}{x}+1\right)$, $f^{-1}(x)=\frac{1}{3}\left(\frac{1}{x}+1\right)$

Q2b Domain of f^{-1} is $R\setminus\{0\}$.

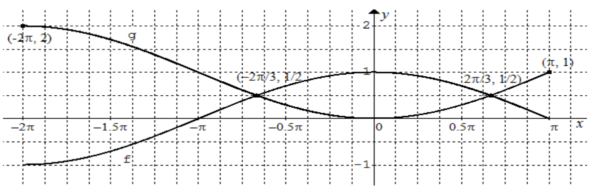
Q2c $c=-\frac{1}{3}$, $d=\frac{1}{3}$

Q3a $\Pr(H)=\frac{2}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{1}{3}=\frac{4}{9}$

Q3b $\Pr(U\mid H)=\frac{\Pr(U\cap H)}{\Pr(H)}=\frac{\frac{2}{3}\times\frac{1}{2}}{\frac{4}{9}}=\frac{3}{4}$

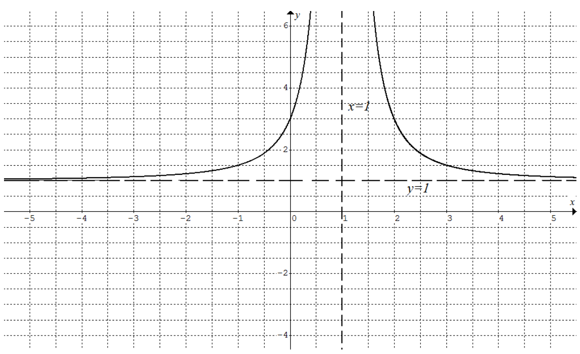
Q4a $\cos\frac{x}{2}=\frac{1}{2}$ and $-\pi\le\frac{x}{2}\le\frac{\pi}{2}$, .: $\frac{x}{2}=-\frac{\pi}{3},\frac{\pi}{3}$, $x=-\frac{2\pi}{3},\frac{2\pi}{3}$

Q4b



Q5ai $f(-1)=\frac{3}{2}$

Q5aii



Q5b Area $=\int_{-1}^{0}\frac{2}{(x-1)^2}+1\,dx=\left[\frac{-2}{x-1}+x\right]_{-1}^{0}=2$

Q6a $\frac{8}{41}$

Q6b $\Pr\left(\hat{P}<\frac{1}{6}\right)=\Pr(X<2)=\Pr(X=0)+\Pr(X=1)$

$=\left(\frac{5}{6}\right)^{12}+12\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{11}=\left(\frac{5}{6}\right)^{11}\left(\frac{5}{6}+2\right)=\frac{17}{6}\left(\frac{5}{6}\right)^{11}$

Q7a $PB=\sqrt{1-x^2}$

Q7b Area $A=\frac{1}{2}(x+1)\sqrt{1-x^2}$

Let $\frac{dA}{dx}=\frac{1}{2}\left(\sqrt{1-x^2}+(x+1)\frac{-x}{\sqrt{1-x^2}}\right)=\frac{1}{2}\left(\frac{1-x^2-x^2-x}{\sqrt{1-x^2}}\right)=0$

.: $2x^2+x-1=0$, $x=\frac{1}{2}$ for max area

$A_{\max}=\frac{1}{2}\left(\frac{3}{2}\right)\left(\frac{\sqrt{3}}{2}\right)=\frac{3\sqrt{3}}{8}$

Q8a $y=ax^2(x+1)(x-1)=ax^2(x^2-1)$, $\left(\frac{1}{\sqrt{2}},1\right)$

.: $a\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)=1$, $a=-4$, .: $f(x)=-4x^2(x^2-1)$

Q8b $x^3+x^2>0$ and $g(x)>0$, .: D is $(-1,1)\setminus\{0\}$

Q8c $h=\log_e\left(\frac{-4x^2(x^2-1)}{x^2(x+1)}\right)=\log_e 4(1-x)$ and $x\in(-1,1)\setminus\{0\}$

As $x\to-1$, $h\to\log_e 8=3\log_e 2$; as $x\to 1$, $h\to-\infty$

As $x\to 0^{\pm}$, $h\to\log_e 4=2\log_e 2$

.: the range of h is $(-\infty,3\log_e 2)\setminus\{2\log_e 2\}$

Q9a $g(f(x))=e^{3+2x-x^2}$

Q9b $\frac{d}{dx}g(f(x))=2(1-x)e^{3+2x-x^2}<0$, .: $1-x<0$, $x>1$

Q9c $f(g(x))=3+2e^x-e^{2x}$

Q9d $3+2e^x-e^{2x}=0$, $(3-e^x)(1+e^x)=0$, $1+e^x\neq 0$

.: $3-e^x=0$, $x=\log_e 3$

Q9e Let $\frac{d}{dx}f(g(x))=2e^x-2e^{2x}=0$, $2e^x(1-e^x)=0$

.: $x=0$ and $y=4$, .: $(0,4)$

Q9f $g(f(x))=e^{3+2x-x^2}>0$ for $x\in R$ and local max is $(1,e^4)$

$f(g(x))=3+2e^x-e^{2x}<0$ for $x>\log_e 3$.

As x increases from $\log_e 3$, $g(f(x))$ is closer to the x-axis and $f(g(x))$ drops further below the x-axis.

.: $g(f(x))+f(g(x))=0$ is possible for one value of $x>\log_e 3$.

Please inform mathline@itute.com re conceptual and/or mathematical errors