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2019 VCAA Math Methods Exam 1 (NHT) Solutions

Q1a $y=\frac{2e^{2x}-1}{e^{x}}=2e^{x}-e^{-x}$

$\frac{dy}{dx}=2e^{x}+e^{-x}=2e^{x}+\frac{1}{e^{x}}=\frac{2e^{2x}+1}{e^{x}}$

Q1b $f(x)=x^{2}\cos 3x$, $f'(x)=2x\cos 3x-3x^{2}\sin 3x$

$f'\left(\frac{\pi}{3}\right)=\frac{2\pi}{3}\cos\pi-3\left(\frac{\pi}{3}\right)^{2}\sin\pi=-\frac{2\pi}{3}$

Q2 $f'(x)=2x^{2}-\frac{1}{4}x^{-\frac{2}{3}}$, $f(x)=\frac{2x^{3}}{3}-\frac{3x^{\frac{1}{3}}}{4}+c$, given $f(1)=-\frac{7}{4}$

.: $\frac{2}{3}-\frac{3}{4}+c=-\frac{7}{4}$, $c=-\frac{5}{3}$.: $f(x)=\frac{2x^{3}}{3}-\frac{3x^{\frac{1}{3}}}{4}-\frac{5}{3}$

Q3a $\int_{2}^{7}\frac{1}{x+\sqrt{3}}\,dx=\left[\log_{e}\left(x+\sqrt{3}\right)\right]_{2}^{7}=\log_{e}\frac{7+\sqrt{3}}{2+\sqrt{3}}=\log_{e}\left(11-5\sqrt{3}\right)$

$\int_{2}^{7}\frac{1}{x-\sqrt{3}}\,dx=\left[\log_{e}\left(x-\sqrt{3}\right)\right]_{2}^{7}=\log_{e}\frac{7-\sqrt{3}}{2-\sqrt{3}}=\log_{e}\left(11+5\sqrt{3}\right)$

Q3b $\frac{1}{2}\left(\frac{1}{x-\sqrt{3}}+\frac{1}{x+\sqrt{3}}\right)=\frac{1}{2}\left(\frac{x+\sqrt{3}+x-\sqrt{3}}{\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)}\right)=\frac{x}{x^{2}-3}$

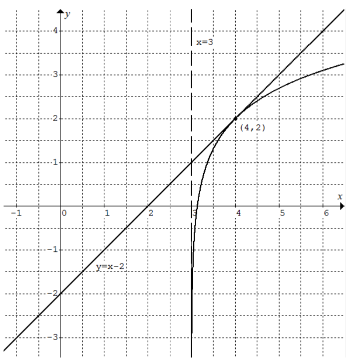
Q3c $\int_{2}^{7}\frac{x}{x^{2}-3}\,dx=\frac{1}{2}\left(\int_{2}^{7}\frac{1}{x-\sqrt{3}}\,dx+\int_{2}^{7}\frac{1}{x+\sqrt{3}}\,dx\right)$

$=\frac{1}{2}\log_{e}\left(11+5\sqrt{3}\right)\left(11-5\sqrt{3}\right)=\frac{1}{2}\log_{e}46$

Q4a Domain: $x-3>0$, $x>3$, $(3,\infty)$; range: R

Q4bi $g'(x)=\frac{1}{x-3}$, $g'(4)=1$, $\frac{y-2}{x-4}=1$, .: $y=x-2$

Q4bii



Q5a $\left(\sqrt{2x+3}-2\right)^{2}=1$, $\sqrt{2x+3}-2=\pm 1$, .: $\sqrt{2x+3}=1$ or 3

.: $2x+3=1$ or 9, $x=-1$ or 3

Q5b $h(x)=\sqrt{2x+3}-2$, range: $[-2,\infty)$

Equation of h^{-1}: $x=\sqrt{2y+3}-2$, $y=\frac{1}{2}(x+2)^{2}-\frac{3}{2}$

$h^{-1}(x)=\frac{1}{2}(x+2)^{2}-\frac{3}{2}$, domain: $[-2,\infty)$

Q6a Consider the next 3 tosses:

$\Pr(2\text{ heads or }3\text{ heads})=\binom{3}{2}\left(\frac{1}{2}\right)^{3}+\left(\frac{1}{2}\right)^{3}=\frac{1}{2}$

Q6b $\hat{p}=\frac{12}{18}=\frac{2}{3}$, $sd\left(\hat{P}\right)\approx\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}=\frac{1}{9}$

90% confid interval $\approx\left(\frac{2}{3}-\frac{33}{20}\times\frac{1}{9},\frac{2}{3}+\frac{33}{20}\times\frac{1}{9}\right)=\left(\frac{29}{60},\frac{17}{20}\right)$

Q7a Horizontal line: $y=\sin a\pi$

$A(a)=\int_{0}^{a}(\sin\pi x-\sin a\pi)\,dx=\left[\frac{-\cos\pi x}{\pi}-x\sin a\pi\right]_{0}^{a}$

$=\frac{1}{\pi}-\frac{1}{\pi}\cos a\pi-a\sin a\pi$

Q7b $1\le a\le\frac{3}{2}$, $A(1)=\frac{1}{\pi}-\frac{1}{\pi}\cos\pi=\frac{2}{\pi}$, $A\left(\frac{3}{2}\right)=\frac{1}{\pi}+\frac{3}{2}$

.: $A(a)\in\left[\frac{2}{\pi},\frac{1}{\pi}+\frac{3}{2}\right]$

Q7ci x-intercept: $y=2\left(\sin\pi x+\frac{\sqrt{3}}{2}\right)=2\sin\pi x+\sqrt{3}=0$,

.: $\sin\pi x=-\frac{\sqrt{3}}{2}$, $x=\frac{4}{3}$

If $a=\frac{4}{3}$, $A(a)=\int_{0}^{\frac{4}{3}}\left(\sin\pi x+\frac{\sqrt{3}}{2}\right)dx$, the required area

$=\int_{0}^{\frac{4}{3}}2\left(\sin\pi x+\frac{\sqrt{3}}{2}\right)dx=2\int_{0}^{\frac{4}{3}}\left(\sin\pi x+\frac{\sqrt{3}}{2}\right)dx=2A(a)$

Q7cii

$\text{Area}=2A(a)=2A\left(\frac{4}{3}\right)=2\left(\frac{1}{\pi}-\frac{1}{\pi}\cos\frac{4\pi}{3}-\frac{4}{3}\sin\frac{4\pi}{3}\right)=\frac{3}{\pi}+\frac{4\sqrt{3}}{3}$

Q8a $\Pr(W=k)=\binom{50}{k}\left(\frac{1}{6}\right)^{k}\left(\frac{5}{6}\right)^{50-k}$

Q8b $\frac{\Pr(W=k+1)}{\Pr(W=k)}=\frac{\binom{50}{k+1}\left(\frac{1}{6}\right)^{k+1}\left(\frac{5}{6}\right)^{49-k}}{\binom{50}{k}\left(\frac{1}{6}\right)^{k}\left(\frac{5}{6}\right)^{50-k}}$

$=\frac{\frac{(50)(49)(48)\cdots(50-k+1)(50-k)}{(k+1)(k)\cdots(1)}\left(\frac{1}{6}\right)}{\frac{(50)(49)(48)\cdots(50-k+1)}{(k)\cdots(1)}\left(\frac{5}{6}\right)}=\frac{(50-k)}{5(k+1)}$

Q5c $\frac{\Pr(W=k+1)}{\Pr(W=k)}=\frac{(50-k)}{5(k+1)}>1$ for $k<\frac{15}{2}$ and $\frac{(50-k)}{5(k+1)}<1$ when $k>\frac{15}{2}$, .: $\Pr(W=k)$ is the greatest when $k=8$.

Please inform mathline@itute.com re conceptual and/or mathematical errors