2020 NSW ESA Mathematics Advanced Solutions

Section I

1	2	3	4	5	6	7	8	9	10
D	B	A	B	C	B	A	A	C	D

Q1 $2x-3\geq 0$, $x\geq \frac{3}{2}$ D

Q2 B

Q3 French $z=1.5$; Commerce $z=3$; Music $z=2$ A

Q4 $\int e+e^{3x}\,dx = ex+\frac{e^{3x}}{3}+c$ B

Q5 TP at $x=\frac{-b}{2a}=\frac{b}{2}>0$ and inverted C

Q6 $[5-2, 5+2]=[3, 7]$ B

Q7 $\int_0^{12} f(x)dx = 3\times 8+\frac{1}{2}\pi 2^2 = 24+2\pi$ A

Q8 Gradient is decreasing, $f''(1)<0$.

Let $f(1)=m$, $0<f'(1)<\frac{m}{1}=f(1)$ A

Q9 $P(W<\mu-\sigma)\approx 0.16$, $0.10<W<0.25$ in the shaded region C

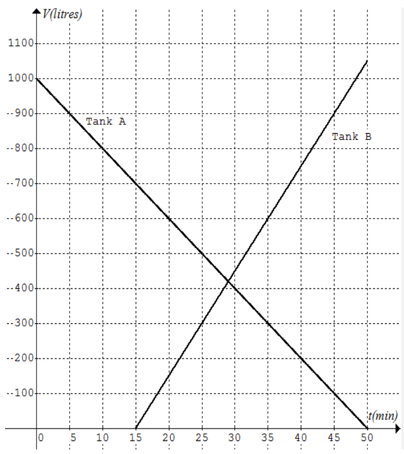
Q10 $h(x)=f(g(x))$, $h'(x)=f'(g(x))\times g'(x)=0$ for stationary pts.

$g'(x)=0$, $x=3$; $f'(g(x))=0$, $g(x)\approx 0.8$, $x\approx 1.8$ or 4.2

Three stationary points D

Section II

Q11a



Q11b Tank A: $V=1000-20t$; Tnak B: $V=30(t-15)$

Same volume: $30(t-15)=1000-20t$, $50t=1450$, $t=29$

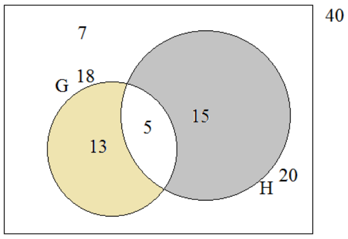
Q11c $30(t-15)+1000-20t=1000$, $t>0$ ∴ $10t=450$, $t=45$

Q12 $a=4$, $d=6$, $l=T_n=4+6(n-1)=1354$, $n=226$

$S_n=\frac{n}{2}(a+l)$, $S_{226}=\frac{226}{2}(4+1354)=153454$

Q13 $\int_0^{\frac{\pi}{4}} \sec^2 x\,dx = [\tan x]_0^{\frac{\pi}{4}} = \tan\frac{\pi}{4}-\tan 0 = 1$

Q14a $P(\text{both})=\frac{5}{40}=\frac{1}{8}$



Q14b $P(H'\mid G)=\frac{P(H'\cap G)}{P(G)}=\frac{\frac{13}{40}}{\frac{18}{40}}=\frac{13}{18}$

Q14c $P\left(H_1H_2'\right)=P(H_1)P\left(H_2'\mid H_1\right)=\frac{20}{40}\times\frac{20}{39}=\frac{10}{39}$

Q15a $100°-35°=65°$

Q15b $AB^2=7^2+9^2-2(7)(9)\cos 65°\approx 76.75$,

$AB\approx\sqrt{76.75}\approx 8.76$ km

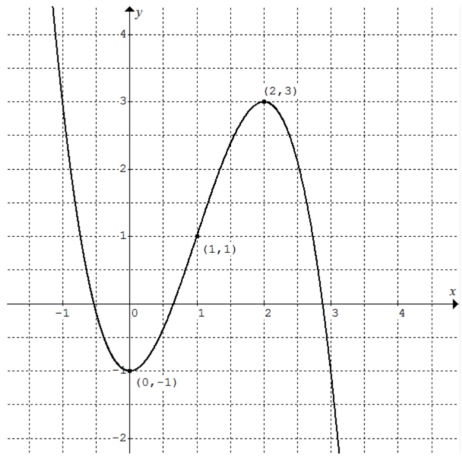
Q15c $\frac{\sin\angle PAB}{9}\approx\frac{\sin 65°}{8.76}$, $\angle PAB\approx 68.61°$

Bearing of B from A $\approx 180°+35°-68.61°\approx 146°$

Q16 y-intercept: $x=0$ and $y=-1$ ∴ $(0,-1)$

Stationary points: $\frac{dy}{dx}=-3x^2+6x=x(-3x+6)=0$ ∴ $x=0, 2$ and $y=-1, 3$ respectively ∴ $(0,-1)$, $(2, 3)$

Inflection point: $\frac{d^2y}{dx^2}=-6x+6=0$ ∴ $x=1$ and $y=1$ ∴ $(1, 1)$



Q17 $\int\frac{x}{4+x^2}dx=\frac{1}{2}\int\frac{du}{u}=\frac{1}{2}\ln|u|+c=\frac{1}{2}\ln\left(4+x^2\right)+c$

Q18a $\frac{d}{dx}e^{2x}(2x+1)=e^{2x}\frac{d}{dx}(2x+1)+(2x+1)\frac{d}{dx}e^{2x}$

$=e^{2x}(2)+(2x+1)2e^{2x}=4(x+1)e^{2x}$

Q18b $\frac{d}{dx}e^{2x}(2x+1)=4(x+1)e^{2x}$ ∴ $\int(x+1)e^{2x}dx=\frac{1}{4}e^{2x}(2x+1)+c$

Q19

$\sec\theta - \cos\theta = \dfrac{1}{\cos\theta} - \cos\theta = \dfrac{1-\cos^2\theta}{\cos\theta} = \dfrac{\sin\theta\sin\theta}{\cos\theta} = \sin\theta\tan\theta$

Q20 Distance

$\approx \dfrac{1}{2}\times\dfrac{1}{60}\left((60+55)+(55+65)+(65+68)+(68+70)+(70+67)\right) \approx 5.4$ km

Q21a $T = 25 + 70(1.5)^{-0.4\times4} \approx 61.6$

Q21b $T = 25 + 70(1.5)^{-0.4t} = 25 + 70e^{(\ln1.5)(-0.4t)} = 25 + 70e^{(-0.4\ln1.5)t}$

$\dfrac{dT}{dt} = (-0.4\ln1.5)70e^{(-0.4\ln1.5)t}$

At $t = 4$, $\dfrac{dT}{dt} = (-0.4\ln1.5)70e^{(-0.4\ln1.5)4} \approx -5.9$

.: cooling rate ≈ 5.9°C min^{-1}

Q21c $T = 25 + 70e^{(-0.4\ln1.5)t} = 55$, $e^{(-0.4\ln1.5)t} = \dfrac{3}{7}$

.: $(-0.4\ln1.5)t = \ln\dfrac{3}{7}$, $t \approx 5.22$

Q22 $AB = 8$, $\angle AOB = 36°$, $\angle OAB = \angle OBA = 72°$

$\dfrac{OA}{\sin72°} = \dfrac{8}{\sin36°}$, $OA = 16\cos36°$,

area $\Delta OAB = \dfrac{1}{2}\times8\times(16\cos36°)\sin72° \approx 49.243$

.: required area ≈ 492.4 cm^2

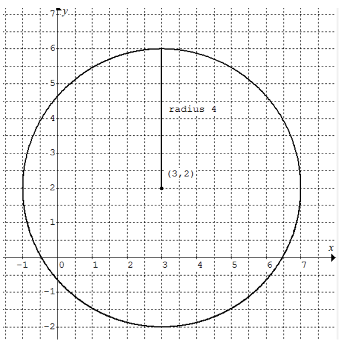
Q23a $\int_0^k \sin x\,dx = 1$, $[-\cos x]_0^k = 1$, $-\cos k + 1 = 1$, $k = \dfrac{\pi}{2}$

Q23b $P(X \le 1) = \int_0^1 \sin x\,dx \approx [-\cos x]_0^1 = -\cos1 + 1 \approx 0.4597$

Q24 $x^2 - 6x + 9 + y^2 + 4y + 4 = 3 + 9 + 4$, $(x-3)^2 + (y+2)^2 = 4^2$

After reflection in the x-axis,

$(x-3)^2 + (y-2)^2 = 4^2$, centre $(3, 2)$ and radius 4.



Q25a Area $= xy + \dfrac{1}{4}\pi x^2 = 36$.: $y = \dfrac{36}{x} - \dfrac{1}{4}\pi x$

$P = x + 2y + x + \dfrac{1}{4}\times2\pi x = 2x + 2y + \dfrac{1}{2}\pi x = 2x + \dfrac{72}{x}$

Q25b $\dfrac{dP}{dx} = 2 - \dfrac{72}{x^2} = 0$, $x = 6$, smallest perimeter $P = 24$

At $x < 6$, $\dfrac{dP}{dx} < 0$; at $x > 6$, $\dfrac{dP}{dx} > 0$.; minimum perimeter at $x = 6$

Q26a $A_1 = 60000(1.005) - 800 = 59500$,

$A_2 = 59500(1.005) - 800 = 58997.50$,

$A_3 = 58997.50(1.005) - 800 \approx 58492.49$ dollars

Q26b $I = (60000 + 59500 + 58997.50)\times0.005 \approx 892.49$ dollars

Q26c $A_{94} = 60000(1.005)^{94} - 800(1.005^{93} + 1.005^{92} + \cdots + 1)$

$A_{94} = 60000(1.005)^{94} - 800\left(\dfrac{1.005^{94}-1}{1.005-1}\right) \approx 187.85$ dollars

Q27 $\bar{x} = 22 - 0.525 = 21.475$, $\bar{y} = \dfrac{684}{20} = 34.2$

$b = \dfrac{y + 10.6063}{x} = \dfrac{\bar{y} + 10.6063}{\bar{x}}$.: $\dfrac{y + 10.6063}{19} = \dfrac{34.2 + 10.6063}{21.475}$

.: $y \approx 29$

Q28a $P(15 < R < 30) \approx 0.475 + 0.34 = 0.815$

$P(N \ge 1) = 1 - P(N = 0) = 1 - (1 - 0.815)^2 \approx 0.9658$

Q28b $P(\text{worker} \cap > 25) = \dfrac{3}{4}\times\dfrac{1}{2} = 0.375$

Q29a $y = c\ln x$, $\dfrac{dy}{dx} = \dfrac{c}{x}$, $\dfrac{dy}{dx} = \dfrac{c}{p}$ at $x = p$ and $y = c\ln p$

Equation of the tangent: $y - c\ln p = \dfrac{c}{p}(x - p)$.: $y = \dfrac{c}{p}x - c + c\ln p$

Q29b $\dfrac{c}{p} = 1$.: $p = c$

$(0, 0)$, $0 = \dfrac{c}{p}(0) - c + c\ln c$.: $\ln c = 1$, $c = e$

Q30a $ax^2 = 4x - x^2$, $(a+1)x^2 - 4x = 0$, $x((a+1)x - 4) = 0$

.: $x = \dfrac{4}{a+1}$

Q30b $A = \int_0^{\frac{4}{a+1}} 4x - x^2 - ax^2\,dx = \dfrac{16}{3}$, $\left[2x^2 - \dfrac{(a+1)x^3}{3}\right]_0^{\frac{4}{a+1}} = \dfrac{16}{3}$

.: $\dfrac{32}{(a+1)^2} - \dfrac{64}{3(a+1)^2} = \dfrac{16}{3}$, $\dfrac{2}{(a+1)^2} = 1$, $a = \sqrt{2} - 1$

Q31a $b = \dfrac{35000 + 5000}{2} = 20000$, $a = 35000 - 20000 = 15000$

Q31b $m(t)$ is increasing for $0 < t < 13$, $c(t)$ is increasing for $10 < t < 36$. Both are increasing for $10 < t < 13$.

Q31c $c(t)$ reaches maximum at $t = 36$ and

$\dfrac{dm}{dt} = 15000\left(\dfrac{\pi}{26}\right)\cos\left(\dfrac{\pi}{26}\times36\right) \approx -643$, i.e. decreasing by 643 per week approximately.

Please inform mathline@itute.com re conceptual and/or mathematical errors.