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Specialist Mathematics

2020

Trial Examination 2
(2 hours)

SECTION A   Multiple-choice questions

Instructions for Section A

Answer **all** questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$

Question 1   For $0 < x \le 1$, $\left|\sqrt{-x} + \dfrac{1}{\sqrt{-x}}\right| =$

A. $\sqrt{x} - \dfrac{1}{\sqrt{x}}$

B. $\dfrac{1}{\sqrt{x}} - \sqrt{x}$

C. $\sqrt{x} + \dfrac{1}{\sqrt{x}}$

D. $\dfrac{1}{\sqrt{-x}} - \sqrt{-x}$

E. $\dfrac{1+\sqrt{x}}{1-\sqrt{x}}$

Question 2   For $\dfrac{\pi}{2} < \theta < \dfrac{3\pi}{2}$, $\operatorname{Arg}(\sin\theta - i\cos\theta) =$

A. $\theta - \dfrac{\pi}{2}$

B. $\theta - \dfrac{3\pi}{2}$

C. $\dfrac{\pi}{2} - \theta$

D. $\dfrac{3\pi}{2} - \theta$

E. $-\theta$

Question 3 Given a, $b \in R^+$, the area of the region bounded by $y = |ax+b| - b$ and $y = b - |ax+b|$ equals

A. $\dfrac{b^2}{2a}$

B. $\dfrac{ab^2}{4}$

C. $\dfrac{2a^2}{b}$

D. $\dfrac{2b^2}{a}$

E. $\dfrac{a}{b^2}$

Question 4 $b\cos^{-1}(x-a) + 2b\sin^{-1}(x-a) =$

A. $b\cos^{-1}(x-a)$

B. $b\sin^{-1}(a-x)$

C. $2b\cos^{-1}(x-a) + b\sin^{-1}(x-a)$

D. $b\sin^{-1}(x-a)$

E. $b\cos^{-1}(a-x)$

Question 5 A particle moves from position A, $\tilde{r}_{\text{A}} = 6\left(\tilde{i} + \sqrt{3}\,\tilde{j}\right)$, to position B, $\tilde{r}_{\text{B}} = \dfrac{5}{2}\left(-\sqrt{3}\,\tilde{i} + \tilde{j}\right)$ in a straight line. The shortest distance of the particle from the origin O is closest to

A. $\dfrac{57}{10}$

B. $\dfrac{58}{11}$

C. $\dfrac{59}{12}$

D. $\dfrac{60}{13}$

E. $\dfrac{61}{14}$

Question 6 $\tilde{a} = \tilde{i} - \tilde{j}$, $\tilde{b} = \tilde{j} + \frac{1}{\sqrt{2}}\tilde{k}$ and unit vector $\tilde{c}$ are linearly independent.

$\tilde{c}$ is possibly be

A. $\tilde{i} + \tilde{j} + \sqrt{2}\,\tilde{k}$

B. $\tilde{i} + \tilde{j} - \sqrt{2}\,\tilde{k}$

C. $\tilde{i} - \tilde{j} + \sqrt{2}\,\tilde{k}$

D. $\frac{\sqrt{3}}{6}\tilde{i} - \frac{\sqrt{3}}{2}\tilde{j} - \frac{1}{\sqrt{6}}\tilde{k}$

E. $-\frac{1}{2}\tilde{i} - \frac{1}{2}\tilde{j} + \frac{\sqrt{2}}{2}\tilde{k}$

Question 7 The position vectors of Particle A and Particle B are $\tilde{r}_A = (t+1)\tilde{i} + t^2\,\tilde{j}$ and $\tilde{r}_B = t^2\,\tilde{i} - t\,\tilde{j}$ respectively for $t \geq 0$. Which one of the following statements is false?

A. A and B move away from each other for $t \geq 0$

B. B is closer to the origin O at any $t \geq 0$

C. The directions of motion of A and B are perpendicular at any $t \geq 0$

D. The particles travel the same distance at $t > 0$

E. The particles travel at the same speed with the same acceleration at $t > 0$

Question 8 $z+1$ and $z-i$ are factors of $f(z) = z^5 + z^4 + az^3 + 5z^2 + bz + c$ where a, b and $c \in R \setminus \{0\}$.

The value of c is

A. 0

B. -1

C. 2

D. -3

E. 4

Question 9 The area of the region in the complex plane enclosed by the three rays $\text{Arg}(z)=\theta+\frac{\pi}{4}$, $\text{Arg}(z)=\theta+\frac{7\pi}{12}$ and $\text{Arg}\left(z-\sqrt{2}-\sqrt{2}\,i\right)=\theta+\frac{11\pi}{12}$ is A. Which one of the following statements is true?

A. $A<\sqrt{3}$ if $0<\theta\le\frac{\pi}{12}$

B. $A>\sqrt{3}$ if $0<\theta\le\frac{\pi}{12}$

C. $A=\sqrt{3}$ if $0<\theta\le\frac{\pi}{12}$

D. $A=\sqrt{3}$ if $-\frac{\pi}{4}<\theta\le 0$

E. $A=\sqrt{3}$ if $-\frac{\pi}{4}<\theta\le\frac{\pi}{12}$

Question 10 Given a, $b\in R^{+}$, if $\sqrt{a+bi}=x+yi$, then $x^2+y^2=$

A. a^2+b^2

B. $2\sqrt{\left|a^2-b^2\right|}$

C. $\sqrt{a^2+b^2}$

D. $\left|a+b\right|^2$

E. $2\left|a-b\right|^2$

Question 11 A particle of mass m is suspended by three strings of zero mass as shown in the diagram below. α, β and γ are angles as defined in the diagram.
$\left|\vec{P}\right|$, $\left|\vec{Q}\right|$ and $\left|\vec{R}\right|$ are respectively the magnitudes of forces $\vec{P}$, $\vec{Q}$ and $\vec{R}$.
Which one of the following statements is false?

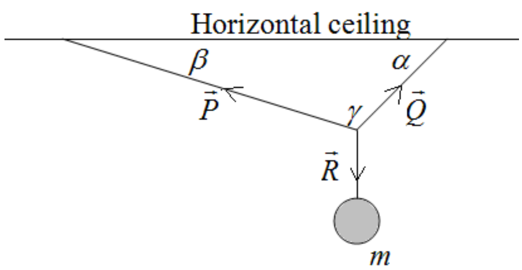
A. $\left|\vec{P}\right|\cos\beta=\left|\vec{Q}\right|\cos\alpha$

B. $\left|\vec{P}\right|\sin\gamma=\left|\vec{R}\right|\cos\alpha$

C. $\left|\vec{Q}\right|\sin\gamma=\left|\vec{R}\right|\sin\beta$

D. $\left|\vec{Q}\right|>\left|\vec{P}\right|$ when $\beta<\alpha$

E. $\vec{P}+\vec{Q}+\vec{R}=\vec{0}$



Question 12 A 2 kg particle slides along a horizontal floor. It slows down at a uniform rate of 3.0 m s^{-1} in a second. The net force (in newtons) exerted by the particle on the floor is closest to

A. 20.5

B. 19.6

C. 15.8

D. 13.6

E. 6.0

Question 13 There is a unit vector which makes the same acute angle $\theta°$ with each of x, y and z axes.

The value of θ is closest to

A. 54.7

B. 53.8

C. 52.9

D. 52.0

E. 51.1

Question 14 A 2.7 kg particle is projected vertically upwards with a speed of u m s^{-1}. It moves under the force of gravity only. It travels the same distance in the third second as in the seventh second. The acceleration due to gravity is constant and has a magnitude of 9.8 m s^{-2}.

The value of u is closest to

A. 40.8

B. 41.9

C. 43.0

D. 44.1

E. 45.2

Question 15 Given a, $b \in R^+$, the tangents to the graphs of $y = b\sin^{-1}\left(\frac{x}{a}\right)$ and $y = b\cos^{-1}\left(\frac{x}{a}\right)$ **cannot** be perpendicular when

A. $b = a$

B. $a > b$

C. $3a > 4b$

D. $3b \le 2a$

E. $2a < b$

Question 16 Given a, $b \in R^+$, the area of the region bounded by the curve $y = b\tan^{-1}\left(\frac{\pi x}{a}\right) + \frac{b\pi}{4}$, the line $x = \frac{a}{\pi}$ and the x-axis is

A. $\frac{ab}{3}$

B. $\frac{ab}{2}$

C. $\frac{2ab}{3}$

D. $\frac{3ab}{4}$

E. $\frac{4ab}{5}$

Question 17 Line $y = mx$ is a tangent to curve $y = \operatorname{cosec}(x)$ at $(p, \operatorname{cosec}(p))$, $p > 0$.
A possible value of p is closest to

A. 1.83

B. 1.98

C. 2.00

D. 2.03

E. 2.37

Question 18 The length L of a metre ruler has a normal distribution with a mean of 100.00 cm and standard deviation of 0.01 cm.
Student A uses a metre ruler to mark 5 continuous sections. Each section is exactly equal to the metre ruler in length.
Student B continues to mark 5 more sections with a second metre ruler. Each section is exactly equal to the second metre ruler in length.

The standard deviation (cm) of the total length of the 10 sections is closest to

A. 0.100

B. 0.071

C. 0.032

D. 0.031

E. 0.030

Question 19 From a random sample taken from a population, an approximate 95% confidence interval for the population mean is determined to be $(1.482, 1.518)$. Another random sample of the same size is taken from the population. Let random variable $\overline{X}$ be the sample mean.

Based on the approximate 95% confidence interval for the population mean $(1.482, 1.518)$, an approximate interval for $\Pr(\overline{X} > 1.475)$ is

A. $(0.85, 1.00)$

B. $(0.78, 1.00)$

C. $(0.65, 0.99)$

D. $(0.58, 0.99)$

E. $(0.58, 0.88)$

Question 20 Using a random sample of 50 items the 95% confidence interval for the population mean is approximately $(x_{\min}, x_{\max})$. Let interval range be $R = x_{\max} - x_{\min}$.

To reduce R by 75% the number of **extra** items required to be sampled randomly is closest to

A. 410

B. 600

C. 750

D. 850

E. 880

SECTION B Extended-answer questions

Instructions for Section B

Answer **all** questions.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this examination are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$

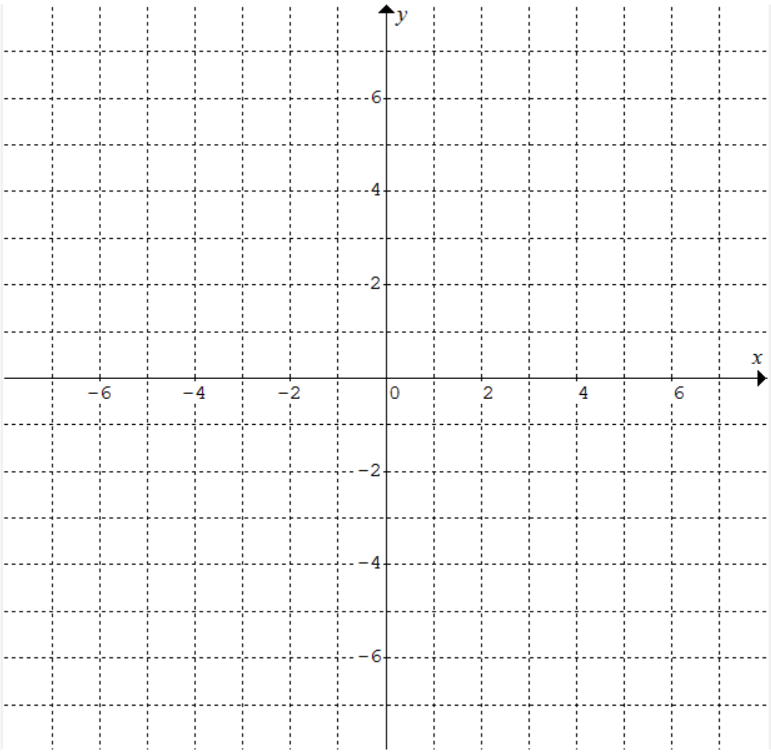
Question 1 $|z+i|-|z-i|=1$ defines the path of a particle in the Argand plane.
Let $x = \text{Re}(z)$ and $y = \text{Im}(z)$ where (x, y) is a point in the Cartesian plane corresponding to z in the Argand plane. Distance is in metres and time in seconds.

a. Show that the Cartesian equation of the path of the particle is $4x^2 - 12y^2 + 3 = 0$. 3 marks

Let $\underset{\sim}{i}$ and $\underset{\sim}{j}$ be unit vectors in the positive direction of x and y axes respectively.
At time $t = 0$, the particle is at $x = 0$ and $y > 0$.

b. In terms of t, $\underset{\sim}{i}$ and $\underset{\sim}{j}$ determine the position vector $\underset{\sim}{r}(t)$ of the particle in the Cartesian plane such that $\dfrac{dx}{dt} = \sqrt{3}$. 3 marks

c. Sketch the graph of the path of the particle for $t \geq 0$.
Show the coordinates of any end points and equations of any asymptotes. 3 marks



d. Find the distance, correct to 2 decimal places, travelled by the particle in the first 2 seconds. 2 marks

e. Find t, correct to 2 decimal places, when the particle travels 5 metres. 1 mark

Question 2 The shape of an upright vessel is formed by rotating the curve $y = \cos^{-1}(1-x)$ about the *y*-axis. Length is measured in metres and time is in seconds.

a. Show that the volume of the vessel is $\frac{3\pi^2}{2}$ m^3. 3 marks

Let h metres be the depth of water in the vessel at time t.

b. Express the volume of water in the vessel in terms of h. 1 mark

The vessel is initially ($t = 0$) full of water. Water flows out at the bottom of the vessel at a rate given by $\frac{dV}{dt} = 0.010h$ m^3 per second.

c. Find the rate of change (with respect to time) of the depth of water in the vessel when the depth is a half of the initial value.

2 marks

d. Find the time taken (correct to 1 decimal place) for the depth to drop to a half of the initial value. Show the definite integral in finding the time. 2 marks

e. Find the average rate of outflow of water (in m^3 per second, correct to 4 decimal places) from the time when outflow starts to the time when it stops.

1 mark

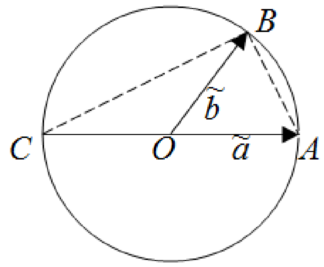
The vessel is initially ($t = 0$) full of water. Water flows out at the bottom of the vessel at a rate given by $\frac{dV}{dt} = 0.010h$ m^3 per second, and water flows into the vessel at the top at 0.005π m^3 per second.

f. Find the minimum depth of water in the vessel. 1 mark

g. Find the time taken (correct to 1 decimal place) for the depth to drop to 75% of the initial value. Show the definite integral in finding the time. 2 marks

Question 3 The following diagram shows a circle with O as its centre. Two vectors $\tilde{a}$ and $\tilde{b}$ from O touch the circumference at A and B. Line segment AC is a diameter of the circle.

a. Express $\overrightarrow{CB}$ and $\overrightarrow{BA}$ in terms of $\tilde{a}$ and $\tilde{b}$. 2 marks



b. Use vector method to show that $\overrightarrow{CB}$ is perpendicular to $\overrightarrow{BA}$. 2 marks

c. Use vector method to show that ΔCBA satisfies Pythagoras Theorem. 2 marks

d. P is a point on line segment CB such that line segments OP and CB are perpendicular. With reference to resolutes show that P is the mid point of line segment CB.

1 mark

Question 4 Distance is in metres, time is in seconds and force is in newtons (N).
Unit vectors $\tilde{i}$ and $\tilde{j}$ point to East and North respectively.
Four horizontal forces act on a 2 kg particle. The particle has an initial velocity of $2\tilde{j}$.
The forces are $1\text{ N }0°\text{T}$, $2\text{ N }30°\text{T}$, $4\text{ N }60°\text{T}$ and $8\text{ N }120°\text{T}$.

a. Show that the acceleration of the particle is $\left(3\sqrt{3}+\frac{1}{2}\right)\tilde{i}+\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)\tilde{j}$. 2 marks

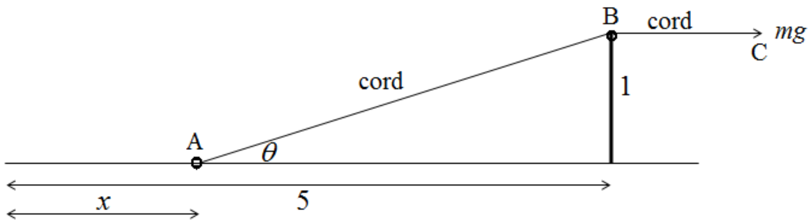
b. Determine the velocity of the particle after 2 seconds. 2 marks

c. Calculate the change in momentum of the particle in the first 2 seconds. 1 mark

d. Calculate the change in speed (correct to 1 decimal place) of the particle in the first 2 seconds. 2 marks

e. Is the motion of the particle linear in the first 2 seconds? Explain. 1 mark

Question 5 A cord (ABC) of zero mass pulls a m kg particle located at A.
The cord runs over a smooth pulley at B. The vertical support of the pulley is 1 metre in height.
The cord makes an angle of θ^c with the horizontal floor.
Initially ($t = 0$) the particle is 5 m from the vertical support and has a speed of 0.1 m s^{-1}.
A constant force of mg newtons pulls the cord at C.
The particle moves along the frictionless horizontal floor and travels x metres after t seconds.
There is a constant resistive force of $0.75mg$ newtons against the motion of the particle.
Assume that the particle and the pulley are dimensionless. $g = 9.8\text{ m s}^{-2}$



Correct numerical answers to 2 decimal places unless stated otherwise.

a. Show that the acceleration (m s^{-2}) of the particle is given by $\left(\dfrac{5-x}{\sqrt{1+(5-x)^2}} - 0.75\right)g$. 3 marks

b. Find x (in m) when the particle starts to slow down. 1 mark

c. Determine the maximum speed (in m s^{-1}) of the particle. 3 marks

d. Determine the time (in s) required to reach the foot of the vertical support. 2 marks

e. The maximum constant resistive force which allows the particle to reach the foot of the vertical support is αmg newtons. Find the value of α.

2 marks

Question 6 Certain brand of spiral compact fluorescent lamps claims to have a mean life of 8500 hours.

For part a to part e, assume normality of the lamp life (X) distribution with mean life of 8500 hours and standard deviation of 1200 hours.

Correct numerical answers to 4 decimal places unless stated otherwise.

a. Find the probability that a lamp can last for a year when it is left on continuously.
Assume 365 days in a year and 24 hours in a day. 1 mark

b. Find the probability that a lamp has a life longer than 9000 hours if it has done 8000 hours. 1 mark

c. A second new lamp is turned on after the first lamp's life ended.
Calculate the mean and standard deviation (correct to the nearest unit) of the total life of the two lamps.

1 mark

d. Consider taking a random sample of 25 such lamps.
Calculate the probability that the mean life of the lamps in the sample is greater than 9000 hours. 1 mark

e. Consider taking three such random samples of 25 lamps, find the probability that at least one sample has a mean life greater than 9000 hours.

1 mark

f. A random sample of 25 lamps has a mean life of 8700 hours. Assuming that the standard deviation of the lamp life is 1200 hours, use the 95% confidence interval to determine whether the brand's claim of 8500 hour mean life is to be accepted or rejected.

2 marks

g. A larger random sample of 250 lamps has a mean life of 8300 hours. Assuming that the standard deviation of the lamp life is 1200 hours, use the 95% confidence interval to determine whether the brand's claim of 8500 hour mean life is to be accepted or rejected.

1 mark

h. Based on the results in part f and part g, explain whether the brand's claim of 8500 hour mean life is to be accepted or rejected.

1 mark

i. State and explain the type of error if any when the brand's claim of 8500 hour mean life is accepted.

1 mark

End of Exam 2

