2020 VCAA Further Mathematics Exam 2 Solutions

SECTION A - Core

Data analysis

Q1a Positively skewed.

Q1b $\frac{24.5+24.6}{2}=24.55$

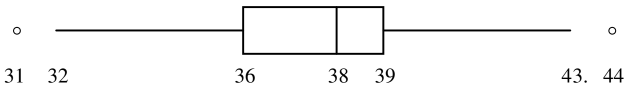
Q1c $\frac{12}{32}\times 100\% = 37.5\%$

Q2a 38

Q2bi $(1-0.997)\times 250 = 0.75 \approx 1$

Q2bii 1

Q2c Lower fence $= 36-1.5\times 3 = 31.5$; upper fence $= 39+1.5\times 3 = 43.5$



Q3a $\frac{50}{250}\times 100\% = 20\%$

Q3b $26.0-23.4=2.6$

Q3c *BMI* 30 or more: Because the *BMI* values are all rounded to one decimal place, there can be one or more men for each circle (outlier) .: there are 4 or more men of the average neck size group, and $\frac{1}{4}\times 76 = 19$ for the above average neck size. Total ≥ 23

Q3d The boxplots show that *BMI* is strongly associated with *neck size*. The larger the *neck size* the greater is the *median BMI*.

Q4ai 24

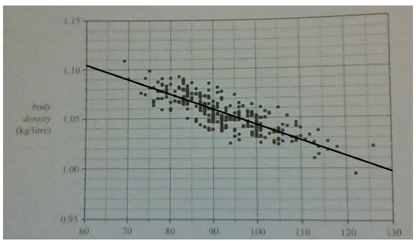
Q4aii 1.065

Q4bi Weight

Q4bii Slope $= -0.00112$

Q4c 29%

Q5a



Q5b Body density $= 1.195-0.001512\times 65 \approx 1.10$ kg/litre

Q5c Extrapolating

Q5d The *body density* decreases by 0.001512 kg/litre for every cm increase in *waist measurement*.

Q5e Residual $= 0.995-1.0101 \approx -0.02$

Q5f $r=-\sqrt{0.6783}\approx -0.824$

Q5g Yes, because the residual plot is random and centred at $residual = 0$. It has no defined pattern.

Q6a Difference in mean heights $= 167.1-156.7 = 10.4$ cm

Q6b The association is strong and negative.

Q6c mean height $= 171+(-0.169)\times$ mean age

Q6d mean height $= 167.9+(-0.001621)\times(\text{mean age})^2$

Recursion and financial modelling

Q7a $15000

Q7b After one year: $V_1 = V_0 - 15000 = 120000-15000 = 105000$
After another year: $V_2 = V_1 - 15000 = 105000-15000 = 90000$ dollars

Q7c Annual flat rate $= \frac{15000}{120000}\times 100\% = 12.5\%$

Q7d $V_n = 120000+(-15000)n$, $n = 0, 1, 2, \cdots$

Q8ai $318718.08-318074.23 = 643.85$ dollars

Q8aii Interest $= 318074.23\times\frac{3.6}{100\times 12}\approx 954.22$
Balance $\approx 318074.23-1600.00+954.22 = 317428.45$ dollars

Q8b $S_0 = 320000.00$, $S_{n+1} = 1.003\times S_n - 1600.00$

Q9a $B_4 = 1.003^4\times 5000 \approx 5060.27$ dollars

Q9b Monthly interest rate $= (1.003-1)\times 100\% = 0.3\%$

Q9c Balance $\approx \$5793$, extra amount $\approx 5793-5183 = 610$ dollars

Q10a After one month $= 1.0024\times 500000-2000 = 499200$
After another month $= 1.0024\times 499200-2000 = 498398.08$ dollars

Q10b $1+\frac{r}{100\times 12} = 1.0024$, $r = 2.88$, annual rate 2.88%

Q10c $500000 = k(500000)-2000$, $k = 1.004$

Q11 36 payments (3 years) of $2400 from $329587.25 reduced to $280875.15.
From $280875.15 it will take 150 more monthly repayments to fully repay the loan.

SECTION B - Modules

Module 1: Matrices

Q1a 1×3

Q1b $\frac{700}{0.25}=2800$ shoppers at Grandmall

Q1ci $Q=P\times V=\begin{bmatrix}0.48\\0.27\\0.25\end{bmatrix}\begin{bmatrix}2300 & 2700 & 2200\end{bmatrix}=\begin{bmatrix} & 1296 & \\ & 729 & \\ & & \end{bmatrix}$

Q1cii q_{23} represents the number of shoppers in clothing at Westmall.

Q1d $\begin{bmatrix}\text{total amount}\end{bmatrix}=\begin{bmatrix}135 & 143 & 131\end{bmatrix}\begin{bmatrix}21.30\\34.00\\14.70\end{bmatrix}=\begin{bmatrix}9663.20\end{bmatrix}$

Q1e $A_{2020}=K\times A_{2019}$, $K=\begin{bmatrix}1.05 & 0 & 0\\0 & 0.85 & 0\\0 & 0 & 0.99\end{bmatrix}$

Q2a Determinant $=1\neq 0$.: the matrix has an inverse.

Q2b $\begin{bmatrix}-7 & \\ -4 & 5\end{bmatrix}$

Q2c $-4\times 7+5\times 6=2$

Q3a $S_1=T\times S_0=\begin{bmatrix}0.80 & 0.09 & 0.10\\0.12 & 0.79 & 0.10\\0.08 & 0.12 & 0.80\end{bmatrix}\begin{bmatrix}250000\\230000\\200000\end{bmatrix}=\begin{bmatrix}240700\\231700\\207600\end{bmatrix}\begin{matrix}W\\G\\E\end{matrix}$

Q3b Difference $=250000-220000=30000$

Q3c $\begin{bmatrix}0.80 & 0.09 & 0.10\\0.12 & 0.79 & 0.10\\0.08 & 0.12 & 0.80\end{bmatrix}^7\begin{bmatrix}250000\\230000\\200000\end{bmatrix}=\begin{bmatrix} \\ 233710\\ \end{bmatrix}\begin{matrix}W\\G\\E\end{matrix}$

Maximum number in the seventh week, 233710.
In the sixth week, 233708; the eighth week, 233689.

Q3d 218884

Q4a $R_3=TR_2+B=\begin{bmatrix}237966\\ \\ \end{bmatrix}\begin{matrix}W\\G\\E\end{matrix}$

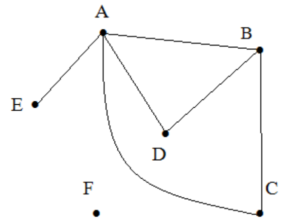
Q4b $R_1=T^{-1}(R_2-B)=\begin{bmatrix}241000\\ \\ \end{bmatrix}\begin{matrix}W\\G\\E\end{matrix}$

Module 2: Networks and decision mathematics

Q1a One (1 edge)

Q1b Dale and Cameron

Q1c



Q2

Player	Batting position
A	3
B	1
C	2

Q3a $0.6+1.2+0.6+0.8=3.2$ km

Q3bi Eulerian trail

Q3bii Exercise station *P*

Q3c This track is between exercise station *S* an exercise station *T*.

Q4a 10 possible routes

Q4b 52

Q4c Minimum cut $20+10+20=50$.: maximum flow is 50.

Q5a From the end of activity *B* to the start of activity *C*.

Q5b Duration of activity *C* is 2 months.

Q5c *A*, *F*, *H* and *E*

Q5d Reduce the completion time of activity *B* from 5 to 2 months so that the project can be completed in minimum time of 17 months.

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Module 3: Geometry and measurement

Q1a $V = Ah = 43 \times 7 = 301\ \text{cm}^3$

Q1b $r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{43}{\pi}} \approx 3.7\ \text{cm}$

Q1c $TSA = 43 + 43 + 2\pi(3.7)(7) \approx 249\ \text{cm}^2$

Q1d $P = 74 \times 4 = 296\ \text{cm}$

Q1e The shelf consists of three squares of $37 \times 37\ \text{cm}^2$.
Each square can fit 100 .: 300 can fit on the shelf.

Q1f $h = 37 - 26\sin 30° = 24\ \text{cm}$

Q2a $A = \frac{1}{2} \times 4.8 \times 4.8 \times \sin 60° \approx 10\ \text{cm}^2$

Q2b $A = \frac{3}{4}(10+10) = 15\ \text{cm}^2$

Q2c 1 : 2

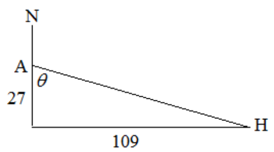
Q2d $4.8 \times 2 = 9.6\ \text{cm}$

Q3a $r = 6400\cos 24° \approx 5847\ \text{km}$

Q3b Distance $= 5847 \times \frac{\pi}{180} \times (90-54) \approx 3674\ \text{km}$

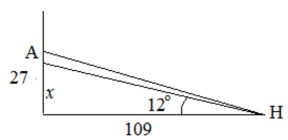
Q3c 1 : 00 pm Dhaka = 11 : 00 am Abu Dhabi
11 hours later, 10 : 00 pm Abu Dhabi

Q3d



$\tan\theta = \frac{109}{27}$, $\theta \approx 76°$, bearing $= 180 - 76 = 104°$

Q3e $282 - 270 = 12$



$x = 109\tan 12° \approx 23$
The hangar is 4 km south of the airport.

Module 4: Graphs and relations

Q1a 8 pm

Q1b Average speed $= \frac{240}{4} = 60\ \text{km/h}$

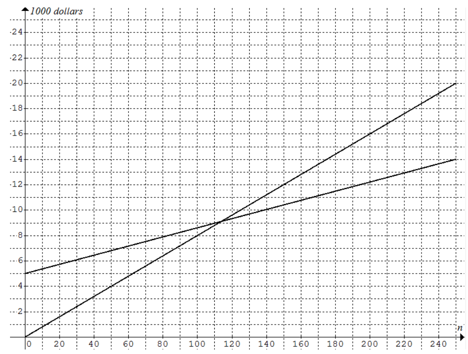
Q1c $d = 0$ when $n = 8$.: $k = 640\ \text{km}$

Q2a Amount $= \$120 \times 15 + \$30 \times 26 = \$2580$

Q2b Let n be the number of children.
$2n + 5(537 - n) = 1644$, $n = 347$

Q2c $k = Fs = 600$, $F = \frac{k}{s} = \frac{600}{80} = 7.5$ km/litre

Q3a



Q3b $80n > 36n + 5000$, $n \geq 114$, at least 114 seat covers

Q3c Number of seat covers $= 180 + 60 = 240$
Let q be the new selling price.
$240q - (36 \times 240 + 5000) = 2920$, $q = 69$ dollars

Q3d $C = mn + p$, $15800 = 300m + p$, $17100 = 350m + p$
.: $m = 26$

Q4a The number of cars that are serviced and detailed each day is 16 or more.

Q4b $\frac{96}{8} = 12$ employees

Q4c Feasible region: $x + 3y \leq 72$, when $x = 20$, $3y \leq 52$
.: maximum number of trucks is 17

Q4d $2x + 3y = 96$ and $x + 3y = 72$ intersect at $(24, 16)$
$2x + 3y = 96$ and $y = 10$ intersect at $(33, 10)$
$2x + 3y = 96$ and $150x + 225y = P$ have the same gradient.
Maximum profit P at points $(24, 16)$, $(27, 14)$, $(30, 12)$ and $(33, 10)$.

Please inform mathline@itute.com re conceptual and/or mathematical errors