2020 VCAA Mathematical Methods Exam 1 Solutions

Q1ai $\frac{dy}{dx} = 2x\sin x + x^2\cos x = x(2\sin x + x\cos x)$

Q1b $f'(x) = (2x-1)e^{x^2-x+3}$, $f'(1) = e^3$

Q2a $\frac{3}{20} - \frac{1}{20} = \frac{1}{10}$

Q2b $\frac{n}{m+n} - \frac{1}{m+n} = 0.05$, $m = \frac{0.95n-1}{0.05} = 19n - 20$

Q3 $\tan(-a+b) = -1$, $-a+b = -\frac{\pi}{4}$; $\tan(a+b) = \sqrt{3}$, $a+b = \frac{\pi}{3}$

$\therefore\ b = \frac{\pi}{24}$, $a = \frac{7\pi}{24}$

Q4 $\log_2 \frac{(x+5)^2}{x+9} = 1$, $x > -5$ and $x > -9$ $\therefore\ x > -5$

$\frac{(x+5)^2}{x+9} = 2$, $x^2 + 8x + 7 = (x+7)(x+1) = 0$, $\therefore\ x = -1$

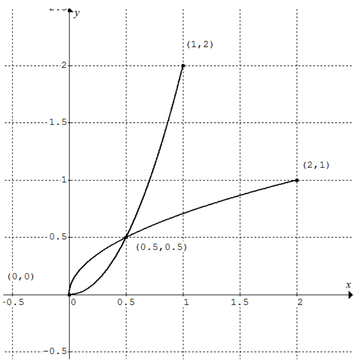
Q5a $\Pr(X \ge 3) = {}^4C_3\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^1 + {}^4C_4\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^0 = \frac{297}{625}$

Q5b $\Pr(X = 2 \mid X \ge 1) = \frac{\Pr(X=2)}{1-\Pr(X=0)} = \frac{6^3}{5^4 - 2^4}$

Q6a $f(x)$: domain $[0, 2]$, range $[0, 1]$

Inverse $x = \frac{1}{\sqrt{2}}\sqrt{y}$, $y = f^{-1}(x) = 2x^2$, domain $[0, 1]$; range $[0, 2]$

Q6b



Q6c $\int_0^{\frac{1}{2}}\left(\frac{1}{\sqrt{2}}\sqrt{x} - 2x^2\right)dx + \int_{\frac{1}{2}}^{1}\left(2x^2 - \frac{1}{\sqrt{2}}\sqrt{x}\right)dx$

$= \left[\frac{\sqrt{2}x^{\frac{3}{2}}}{3} - \frac{2x^3}{3}\right]_0^{\frac{1}{2}} + \left[\frac{2x^3}{3} - \frac{\sqrt{2}x^{\frac{3}{2}}}{3}\right]_{\frac{1}{2}}^{1} = \frac{5-2\sqrt{2}}{6}$

Q7a $f(1) = 9 \ne 0$ $\therefore\ P(1, 0)$ is not on the graph of $y = f(x)$.

Q7bi Slope $PQ = \frac{f(a)-0}{a-1} = \frac{a^2+3a+5}{a-1}$

Q7bii $f'(x) = 2x+3$, $f'(a) = 2a+3$

Q7biii Tangent: $y = f'(a)x + c$, $f(a) = f'(a)a + c$ $\therefore\ c = 5 - a^2$

$\therefore\ y = (2a+3)x + 5 - a^2$ and $(1, 0)$ $\therefore\ a^2 - 2a - 8 = (a-4)(a+2) = 0$

$\therefore\ a = -2,\ 4$

Q7biv Choose $a = 4$, $y = 11x - 11$ OR choose $a = -2$, $y = -x + 1$

Q7c $f'(x) = 2x + 3 = 0$, $x = -\frac{3}{2}$ at the turning point.

Translate $f(x)$ so that the turning point is above point $P(1, 0)$

$\therefore\ k = \frac{3}{2} + 1 = \frac{5}{2}$

Q8a $f'(x) = 1 + \log_e x = 0$, $x = \frac{1}{e}$, $y = f\left(\frac{1}{e}\right) = -\frac{1}{e}$ $\therefore\ Q\left(\frac{1}{e}, -\frac{1}{e}\right)$

Q8b $x\log_e x = \frac{1}{2}\left(\frac{d}{dx}x^2\log_e x - x\right)$

$\int x\log_e x\,dx = \frac{1}{2}\int\left(\frac{d}{dx}x^2\log_e x - x\right)dx = \frac{x^2\log_e x}{2} - \frac{x^2}{4}$

Q8c $b\log_e b = 0$ $\therefore\ b = 1$

Area $= -\int_{\frac{1}{e}}^{1} x\log_e x\,dx = -\left[\frac{x^2\log_e x}{2} - \frac{x^2}{4}\right]_{\frac{1}{e}}^{1} = \frac{1}{4}\left(1 - \frac{3}{e^2}\right)$

Q8di $g(x) = x\log_e x + k$, $g'(x) = 1 + \log_e x$

Let $g'(x) = 1 + \log_e x = 2$, $x = e$ and $y = 2x = 2e$

$\therefore\ g(e) = e + k = 2e$ $\therefore\ k = e$

Q8dii Let $g'(x) = 1 + \log_e x = 1$ $\therefore\ x = 1$ and $y = x = 1$

$\therefore\ g(1) = k = 1$

Do not intersect, $k > 1$

Please inform mathline@itute.com re conceptual and/or mathematical errors