2020 VCAA Specialist Mathematics Exam 1 Solutions

Q1a $R+5\sin 30°+10\sin 60°-2g=0$, $R=2g-\dfrac{5(1+2\sqrt{3})}{2}$

Q1b $2a=10\cos 60-5\cos 30=\dfrac{5(2-\sqrt{3})}{2}$, $a=\dfrac{5(2-\sqrt{3})}{4}$

Q1c Distance $=\dfrac{1}{2}\times\dfrac{5(2-\sqrt{3})}{4}\times 4^2=10(2-\sqrt{3})$

Q2 $u=1-x$, $\dfrac{du}{dx}=-1$,

$$\int_{-1}^{0}\frac{1+x}{\sqrt{1-x}}dx=\int_{2}^{1}-\frac{2-u}{\sqrt{u}}du=\int_{1}^{2}\frac{2-u}{\sqrt{u}}du=\int_{1}^{2}2u^{-\frac{1}{2}}-u^{\frac{1}{2}}du$$

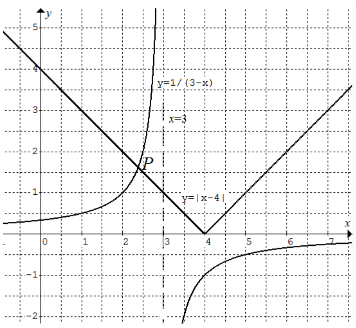
$$=\left[4u^{\frac{1}{2}}-\frac{2u^{\frac{3}{2}}}{3}\right]_1^2=\frac{8}{3}\sqrt{2}-\frac{10}{3}$$

Q3 $z^3=\dfrac{1}{\sqrt{2}}-\dfrac{1}{\sqrt{2}}i=\text{cis}\left(\dfrac{7\pi}{4}+2k\pi\right)=\text{cis}\left(\dfrac{(7+8k)\pi}{4}\right)$

$z=\text{cis}\left(\dfrac{(7+8k)\pi}{12}\right)$ Let $k=-1,0,1$, $z=\text{cis}\left(-\dfrac{\pi}{12}\right)$, $\text{cis}\left(\dfrac{7\pi}{12}\right)$

and $\text{cis}\left(\dfrac{15\pi}{12}\right)=\text{cis}\left(-\dfrac{3\pi}{4}\right)$

Q4 $3-x>\dfrac{1}{|x-4|}$.: $3-x>0$.: $x<3$ and $|x-4|>\dfrac{1}{3-x}$



P: $4-x=\dfrac{1}{3-x}$, $x^2-7x+11=0$, $x=\dfrac{7-\sqrt{5}}{2}$

.: $x\in\left(-\infty,\dfrac{7-\sqrt{5}}{2}\right)$

Q5a $\hat{b}=\dfrac{\tilde{i}+m\tilde{j}-\tilde{k}}{\sqrt{2+m^2}}$, $\tilde{a}.\hat{b}\,\hat{b}=\dfrac{1-3m}{\sqrt{2+m^2}}\times\dfrac{\tilde{i}+m\tilde{j}-\tilde{k}}{\sqrt{2+m^2}}$

$\dfrac{1-3m}{2+m^2}=-\dfrac{11}{18}$ and m is an integer, $(11m-10)(m-4)=0$.: $m=4$

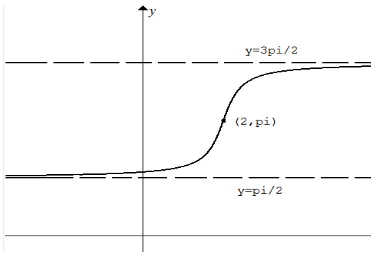
Q5b Component perpendicular to $\tilde{b}$:

$2\tilde{i}-3\tilde{j}+\tilde{k}-\left(-\dfrac{11}{18}\tilde{i}-\dfrac{44}{18}\tilde{j}+\dfrac{11}{18}\tilde{k}\right)=\dfrac{47}{18}\tilde{i}-\dfrac{5}{9}\tilde{j}+\dfrac{7}{18}\tilde{k}$

Q6a $f'(x)=\dfrac{3}{1+(3x-6)^2}=\dfrac{3}{9x^2-36x+37}$

Q6b $f''(x)=\dfrac{-3(18x-36)}{(9x^2-36x+37)^2}$, $f''(x)=0$ at the inflection pt .: $x=2$

Q6c



Q7a $f(x)=\begin{cases}mx+n & x<1\\ \dfrac{4}{1+x^2} & x\ge 1\end{cases}$ $\quad f'(x)=\begin{cases}m & x<1\\ \dfrac{-8x}{(1+x^2)^2} & x\ge 1\end{cases}$

.: at $x=1$, $m+n=\dfrac{4}{2}=2$ and $m=\dfrac{-8}{4}=-2$.: $n=4$

Q7b $x=0$, $y=4$; $x=1$, $y=2$

Area $=\dfrac{1}{2}(4+2)(1)+\displaystyle\int_1^{\sqrt{3}}\frac{4}{1+x^2}dx=3+4\left[\tan^{-1}x\right]_1^{\sqrt{3}}$

$=3+4\left(\tan^{-1}\sqrt{3}+\tan^{-1}1\right)=3+4\left(\dfrac{\pi}{3}-\dfrac{\pi}{4}\right)=3+\dfrac{\pi}{3}$

Q8 $V=\displaystyle\int_0^{\sqrt{3}}\pi y^2\,dx=4\pi\int_0^{\sqrt{3}}\frac{x^2+x+1}{(x+1)(x^2+1)}dx$

$=4\pi\displaystyle\int_0^{\sqrt{3}}\frac{x^2+x+1}{(x+1)(x^2+1)}dx$ partial fractions

$=2\pi\displaystyle\int_0^{\sqrt{3}}\frac{1}{x+1}+\frac{x}{x^2+1}+\frac{1}{x^2+1}dx$

$=2\pi\left[\log_e(x+1)+\dfrac{1}{2}\log_e(x^2+1)+\tan^{-1}x\right]_0^{\sqrt{3}}$

$=2\pi\left(\log_e(2\sqrt{3}+2)+\dfrac{\pi}{3}\right)$

Q9a $x=\sin^{-1}t$, $\dfrac{dx}{dt}=\dfrac{1}{\sqrt{1-t^2}}$

$y=\log_e(1+t)+\dfrac{1}{4}\log_e(1-t)$; $\dfrac{dy}{dt}=\dfrac{1}{1+t}-\dfrac{1}{4(1-t)}$

$\left(\dfrac{dy}{dt}\right)^2=\dfrac{1}{(1+t)^2}+\dfrac{1}{-2(1-t)(1+t)}+\dfrac{1}{16(1-t)^2}$.: $a=1$, $b=-2$, $c=16$

Q9b

$$s=\int_0^{\frac{1}{2}}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt=\int_0^{\frac{1}{2}}\sqrt{\frac{1}{1-t^2}+\frac{1}{(1+t)^2}-\frac{\frac{1}{2}}{1-t^2}+\frac{1}{16(1-t)^2}}\,dt$$

$$=\int_0^{\frac{1}{2}}\sqrt{\frac{1}{(1+t)^2}+\frac{\frac{1}{2}}{1-t^2}+\frac{1}{16(1-t)^2}}\,dt=\int_0^{\frac{1}{2}}\sqrt{\left(\frac{1}{1+t}+\frac{1}{4(1-t)}\right)^2}\,dt$$

$$=\int_0^{\frac{1}{2}}\frac{1}{1+t}+\frac{1}{4(1-t)}dt=\left[\log_e(1+t)-\frac{1}{4}\log_e(1-t)\right]_0^{\frac{1}{2}}=\log_e\frac{3}{2}+\frac{1}{4}\log_e 2$$

Please inform mathline@itute.com re conceptual and/or mathematical errors.