



Online & home tutors Registered business name: itute ABN: 96 297 924 083

2021
Further
Mathematics

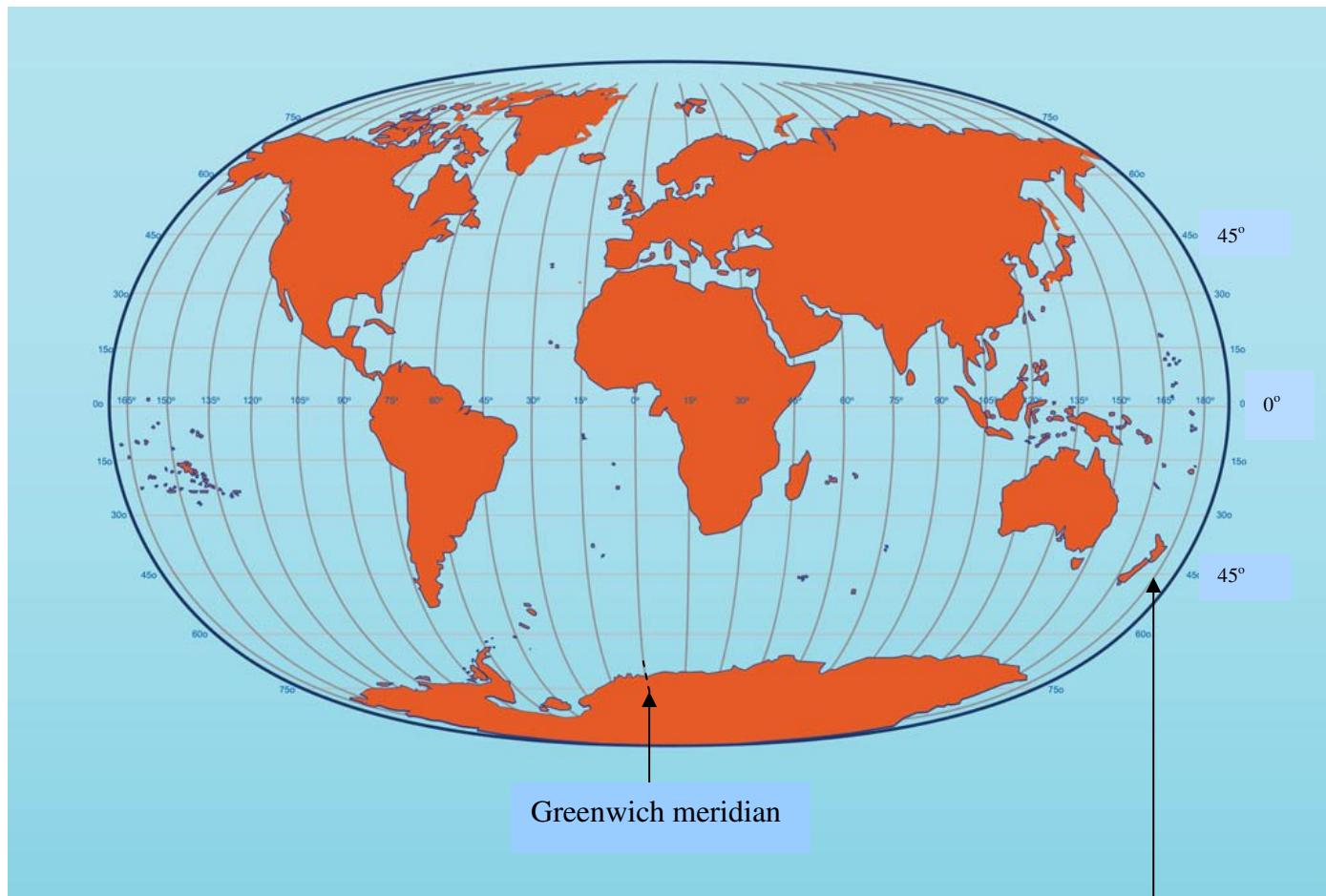
Year 12
Problem Solving Task
Module: Geometry and
measurement

Time allowed: 2 hours plus

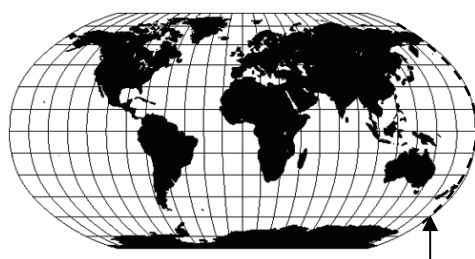
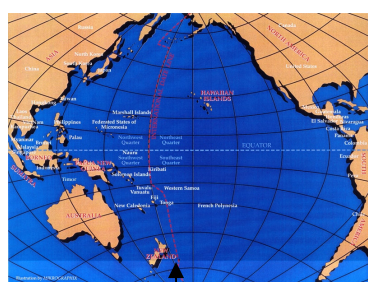
Theme: Travel around the world

Assumed Knowledge: Measurement, geometry and trigonometry including solving right and non-right-angled triangles, three-figure bearings, surface area; spherical geometry including arc length, latitude and longitude, time zones and travel times

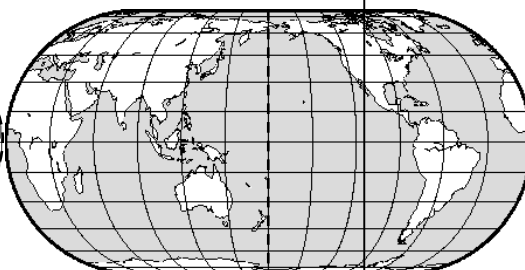
Four world maps with meridians (longitudes) and parallels (latitudes) are shown below.



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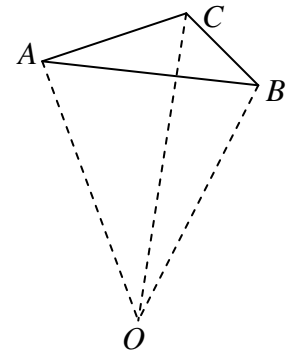
International Date Line

Part I (60 minutes plus)

Correct answers to 2 decimal places unless stated otherwise.

O, A, B and C are four points in 3 dimensional space such that $OA = OB = OC = 10$ units, $BC = 3$ units, $\angle AOB = 60^\circ$ and $\angle ACB = 90^\circ$.

a. Calculate the lengths of AB and AC in units.



b. Calculate $\angle BOC$, $\angle BAC$ and $\angle OAC$. Express your answers in decimal degrees.

c. If O is the centre of a sphere and points A, B and C are on the surface of the sphere, calculate the length of the shortest arc on the surface of the sphere joining (i) B and C , (ii) C and A and (iii) A and B .

d. **Estimate** the area (unit^2) of the shape bounded by arcs BC, CA and AB **on the surface of the sphere**. To estimate the area, use Heron's formula as an approximation $A = \sqrt{s(s-a)(s-b)(s-c)}$ where a, b and c are the side lengths and $s = \frac{a+b+c}{2}$.

Consider the Earth as a perfect sphere of radius 6400 km and take the following journey: Start from the North Pole, 90° N, and travel south along the Greenwich meridian to the equator, then travel along the equator to longitude 60° E, and finally travel north back to the North Pole.

e. Calculate the distances (km) travelled in the first and second parts of the journey.

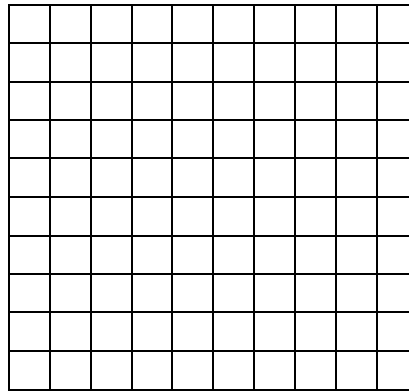
f. The three parts of the journey form a 'triangle' on the surface of the Earth. Calculate the sum of the interior angles of the triangle.

g. Calculate the area (km^2) on the surface of the Earth enclosed by the three parts of the journey.

h. Consider the following journey: Start from the North Pole and travel south along the Greenwich meridian $\frac{2}{3}$ of way towards the equator, then travel on the same latitude to longitude 60° E, and finally travel north back to the North Pole.
Calculate the total distance (km) for the journey.

The Bermuda triangle has vertices at Miami, Florida (26 N, 80 W), San Juan, Puerto Rico (18 N, 66 W) and Bermuda (32 N, 65 W).

i. Plot the three vertices on the following grid. The vertical lines and horizontal lines represent longitudes and latitudes respectively on a plane.



j. **Estimate** the 3-figure bearings of Miami and San Juan from Bermuda. Show your method.

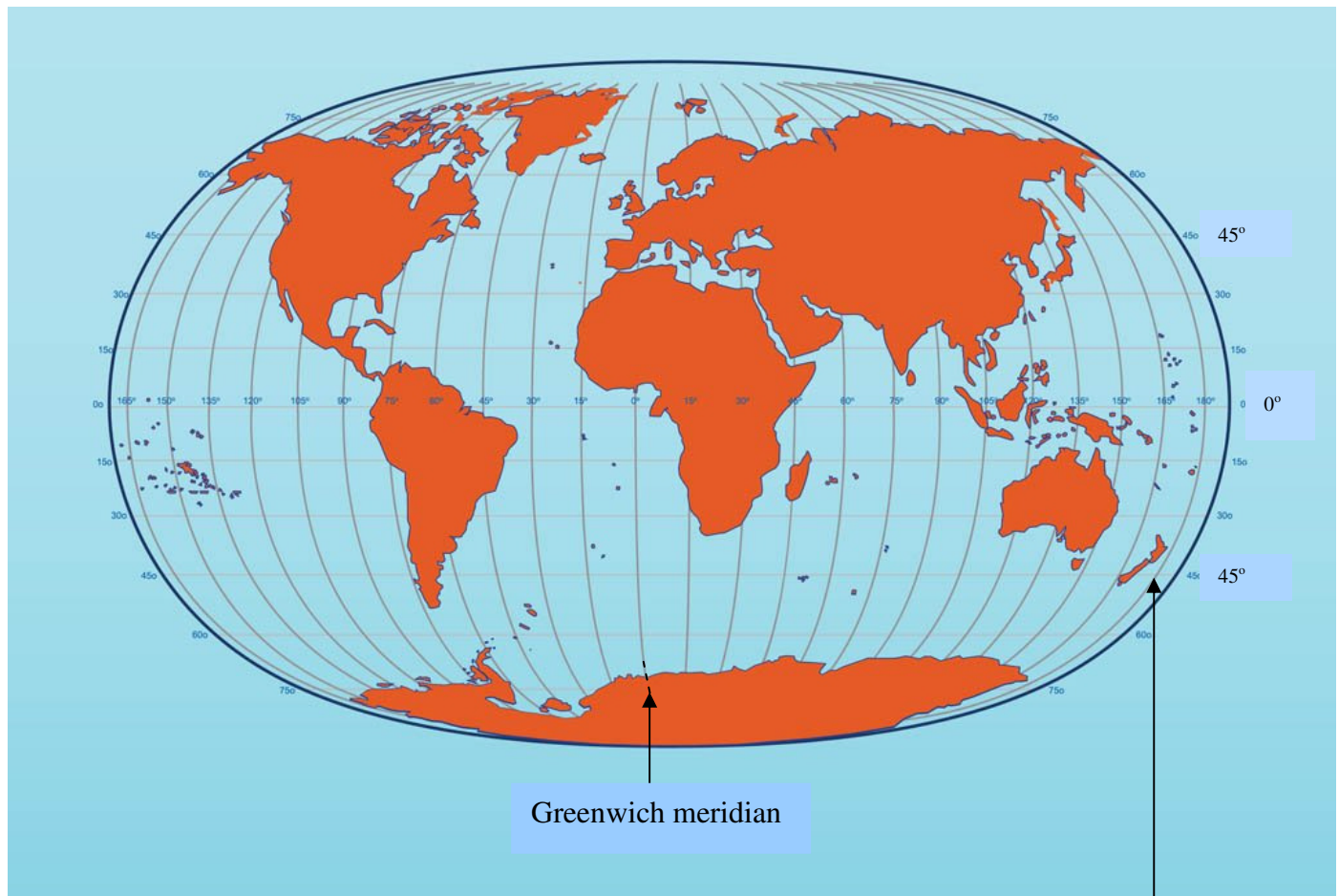
k. **Estimate** the 3-figure bearing of Miami from San Juan. Show your method.

l. **Estimate** the area (km^2) enclosed by the Bermuda triangle, correct to one significant figure.

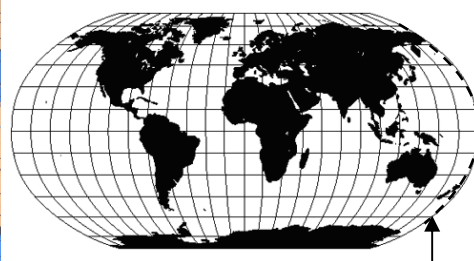
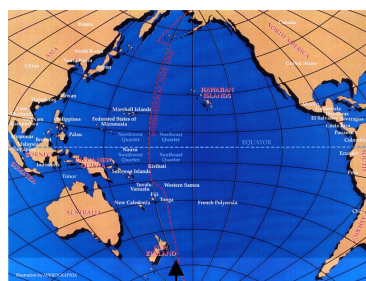
End of Part I

Theme: Travel around the world

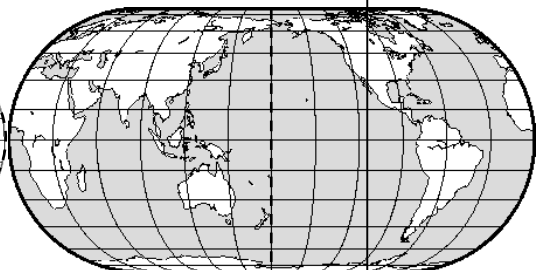
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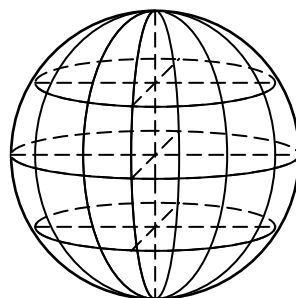
International Date Line

Part II (60 minutes plus)

Consider the Earth as a perfect square of radius 6400 km.

Correct distance to the nearest km and time to the nearest hour unless stated otherwise.

Some locations: **Buenos Aires** Argentina (34° S, 58° W), **Cape Town** South Africa (34° S, 18° E), **Adelaide** Australia (34° S, 139° E), **Tokyo** Japan (35° N, 139° E), **Stockholm** Sweden (59° N, 18° E), **Bangui** Central Africa Republic (5° N, 18° E), **Colombo** Sri Lanka (5° N, 80° E), **Bogota** Colombia (5° N, 74° W)



- a. Calculate the shortest distance between Cape Town (34° S, 18° E) and Stockholm (59° N, 18° E).
- b. A plane flies at 12 km above the Earth surface.
Calculate the extra distance of the direct flight path between Cape Town and Stockholm.
- c. Calculate the radius of the circle formed by Latitude 34° S.
- d. Calculate the distance along Latitude 34° S from Cape Town (34° S, 18° E) to Adelaide (34° S, 139° E).
- e. Calculate the shortest distance from Cape Town to Adelaide.
- f. Calculate the shortest distance from Buenos Aires (34° S, 58° W) to Adelaide (34° S, 139° E).

g. Calculate the shortest distance from Stockholm (59°N , 18°E) to Bangui (5°N , 18°E) and then to Colombo (5°N , 80°E).

h. Calculate the shortest distance from Stockholm (59°N , 18°E) directly to Colombo (5°N , 80°E).

The following questions use 15° of longitude as equating to a 1 hour time difference between two locations.

i. Calculate the time difference between Tokyo (35°N , 139°E) and Bogota (5°N , 74°W).

j. Determine Bogota local time when Tokyo local time is 9 pm Saturday.

k. Determine Tokyo local time when Bogota local time is 9 pm Saturday.

l. A trip from Tokyo travelling **west** to Bogota takes 32 hours.
The local time of departure from Tokyo is 9 pm Saturday. Find the local time of arrival at Bogota.
Show working/reasoning

m. A trip from Tokyo travelling **east** crossing the International Date Line to Bogota takes 30 hours.
The local time of departure from Tokyo is 9 pm Saturday. Find the local time of arrival at Bogota.
Show working/reasoning

End of Part II End of Problem Solving Task