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Specialist Mathematics

2021

Trial Examination 1
(1 hour)

Instructions

Answer **all** questions. Do **not** use calculators.
Unless otherwise specified, an **exact** answer is required to a question.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
In questions where more than one mark is available, show appropriate working or explanation.
Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$

Question 1 (3 marks)
A 1.0 kg mass is at rest on a smooth floor.
The mass is then acted on by 3 horizontal forces: 2 newtons NE, $\left(\sqrt{3}+\sqrt{2}\right)$ newtons W and $\left(\sqrt{2}+1\right)$ newtons S.

a. Find the acceleration of the 1.0 kg mass. Use compass bearing to indicate direction. 2 marks

b. Find the momentum (magnitude and direction) of the mass after 5.0 seconds. 1 mark

Question 2 (3 marks)
The position of a particle is given by $\tilde{r}(t) = (1+\cos t)\tilde{i} + 2t\,\tilde{j}$ and $0 \le t \le \pi$.

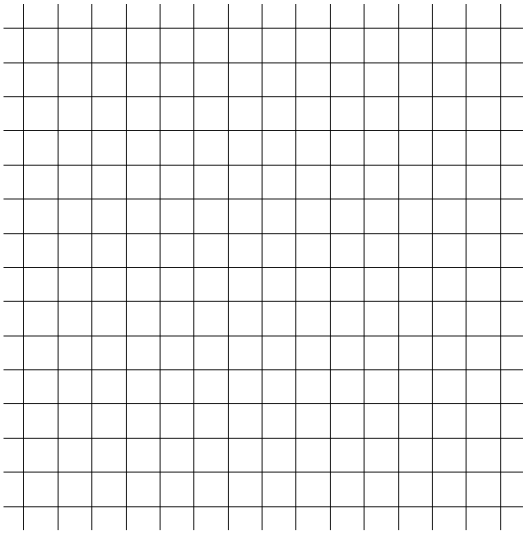
a. Find the maximum and minimum speeds of the particle. 2 marks

b. Find the direction of motion of the particle when it is at its maximum speed.
Give your answer as a unit vector. 1 mark

Question 3 (6 marks)
Consider $f(x) = |\sin^{-1}(x)|$ and $g(x) = \sin^{-1}|x|$.

a. On the grid diagram below sketch the graph of $g(x) = \sin^{-1}|x|$ and then use the method of *addition of ordinates* to sketch $h(x) = |\sin^{-1}(x)| + \sin^{-1}|x|$.

3 marks



bi. Find the area of the region bounded by $g(x) = \sin^{-1}|x|$ and the x-axis. 2 marks

bii. Hence find the area of the region bounded by $h(x) = |\sin^{-1}(x)| + \sin^{-1}|x|$ and the x-axis. 1 mark

Question 4 (3 marks)
Solve $z^4 - z^2 + 1 = 0$ for z. 3 marks

Question 5 (5 marks)

Let $z = 1 + \sqrt{2x - 1 - x^2}$ and $x \in R$.

a. Show that $|z| = \sqrt{2 - 2x + x^2}$. 2 marks

b. Find the asymptotes of $y = |z|$. 2 marks

c. Find the minimum value of $|z|$ without using calculus. 1 mark

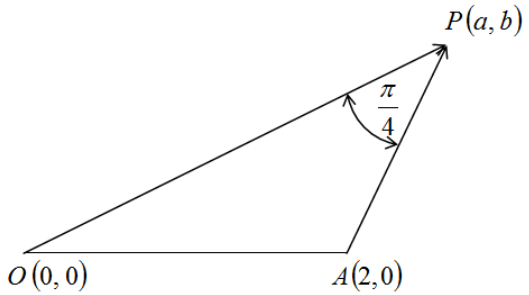
Question 6 (5 marks)

a. Find $\int \frac{dx}{\sqrt{(1 - x^2)\arcsin(x)}}$ 2 marks

b. Evaluate $\int_0^1 \left(\frac{x^3 - x^2 + x}{x^3 + 1} \right) dx$ 3 marks

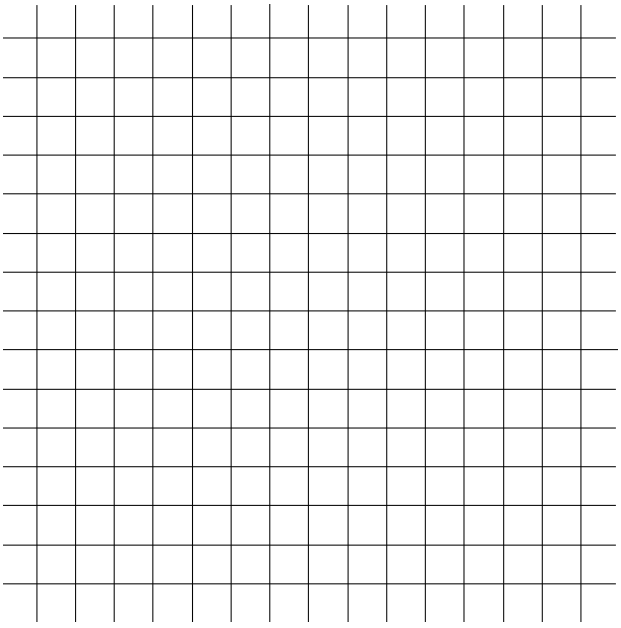
Question 7 (6 marks)

Point $P(a, b)$ moves such that the angle between vectors $\overrightarrow{OP}$ and $\overrightarrow{AP}$ is $\frac{\pi}{4}$ radians, and $a, b \in R$.



a. Find the path(s) of point $P(a, b)$, i.e. the relationship(s) between a and b. 2 marks

b. Sketch the graph of the path(s) of point $P(a, b)$.
Use the same scale on both axes. 4 marks



Question 8 (4 marks)
A very large tank (assume infinite capacity) has V_0 litres of water initially. Solution containing a grams of salt per litre flows into the tank at b litres per minute.

a. Determine the concentration C (grams per litre) of the salt solution in the tank at time t. 2 marks

b. Write a differential equation for the rate of change in concentration of the salt solution. 1 mark

c. Find t in terms of V_0 and b when the concentration of the salt solution in the tank is 90% of the concentration of the salt solution flowing into the tank.

1 mark

Question 9 (5 marks)
Factory A produces bolts of mean radius 1.00 cm and standard deviation 0.01 cm. The length L of a bolt is 4 times its diameter D. During production a sample of 400 bolts is taken.

a. Find the mean and standard deviation of the sum S of length and diameter of a bolt, $S = L + D$. 2 marks

b. Find $\text{E}(\overline{S})$ and $\text{sd}(\overline{S})$ where $\overline{S}$ is the sample mean of S. 2 marks

c. Factory B also produces the same bolts.
During production a sample of 144 bolts is taken and an approximate 95% confidence interval for the mean of S is found to be $(9.99, 10.01)$. Find $\text{sd}(S)$ of bolts produced by Factory B, correct to 2 decimal places.

1 mark

End of Exam 1