2021 VCAA Further Mathematics Exam 2 Solutions

SECTION A - Core

Data analysis

Q1a Results for high jump, short put and javelin for the athletes are numerical variables. 3

Q1b Mean for high jump 1.81

Q1c $z = \dfrac{14.50 - 13.74}{1.43} \approx 0.5$

Q1d $100\% - 16\% = 84\%$

Q1e Because $Q_3 = \max$ of the data set

Q1f Upper fence $= Q_3 + 1.5 \times IQR = 42.88 + 1.5(42.88 - 40.88) = 45.88$
Min distance $= 45.88$ m

Q2a $time800 = 0.0393 + 5.28 \times time200$

Q2b $b = r\dfrac{s_y}{s_x}$, $5.2756 = r\dfrac{8.2910}{0.96956}$, $r \approx 0.616935$,
Coefficient of determination $r^2 \approx 0.38 = 38\%$

Note: $a = \bar{y} - b\bar{x} \approx 6.01468$ inconsistent with $a \approx 0.03931$ in the least squares line.

Q3a Explanatory variable: *year*

Q3b $r \approx -0.938$

Q3c Average decrease $=$ slope $= \dfrac{54 - 60}{2000 - 1960} = -0.15$ seconds/year

Q3d Residual $= 53.83 - 54.10 = -0.27$ seconds

Q3ei $357.1 - 0.1515 \times 2032 = 49.252$

Q3eii Assumption: The decreasing trend (0.15 seconds per year) continues into the future till 16 years after 2016.

Q4a Use the equation of the least squares line for women;
Year 1908, $time \approx 75.26$; year 2020, $time \approx 48.04$
Draw line connecting $(1908, 75.26)$ and $(2020, 48.04)$.

Q4b $difference = (538.9 - 0.2430 \times 2024) - (356.9 - 0.1544 \times 2024)$
≈ 2.7 seconds

Q4c $538.9 - 0.2430 \times year < 356.9 - 0.1544 \times year$
$year > 2054.2$, Olympic 2056

Q5a $\dfrac{1}{dfference} = -2.234 + 0.001209 \times year$ or

$$difference = \frac{1}{-2.234 + 0.001209 \times year}$$

Q5b $difference \approx 4.5$ in year 2032

Recursion and financial modelling

Q6a $1890 \times 12 = 22680$ dollars

Q6b \$420 000

Q6c $S_0 = 420000$, $1 + \dfrac{5.4}{1200} = 1.0045$, $S_{n+1} = 1.0045 \times S_n - 1890$

Q7a Number of cups $= \dfrac{1440}{0.05} = 28800$

Q7b Flat rate $\dfrac{1440}{12000} \times 100\% = 12\%$ pa

Q7c Note: Different definitions for M_n in the introduction and this part.
$M_n = 12000 + (-0.05) \times n$

Q8a $S_1 = 1.001S_0 - 1193 = 569377$ dollars

Q8b $\dfrac{r}{26 \times 100} = 0.001$, $r = 2.6$, compound interest rate $= 2.6\%$

Q8c Number of payments ≈ 650.0046891, use $N = 650$ to find
$FV = -5.59$, final payment $\approx 1193 + 5.59 = 1198.59$ dollars

Q9a Interest rate per month $= \dfrac{5.1}{1200} = 0.00425$
$V_{n+1} = 1.00425V_n - 900$, $V_0 = 152431$
$V_1 = 1.00425 \times 152431 - 900 \approx 152178.8318$
$V_2 = 1.00425 \times 152178.8318 - 900 \approx 151925.59$ dollars

Q9b Amount after 2 years: \$146073.7405
Amount required: \$153112.9399
Extra amount required $153112.9399 - 146073.7405 \approx 7039.20$ dollars

SECTION B - Modules

Module 1: Matrices

Q1a 3×1

Q1b $k=1+\dfrac{5}{100}=1.05$

Q2a Brie and Dex

Q2b $E\to D\to B\to C$

Q2c $A\to B\to D$, $A\to E\to D$

Q3a $8000-\left(0.85\times3200+0.80\times2000+0.90\times2800\right)=1160$

Q3b $S_1=T\times S_0=\begin{bmatrix}3060\\1900\\3040\end{bmatrix}$

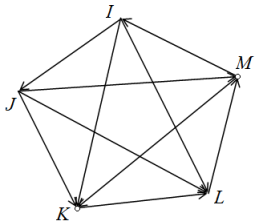
Q3c $\dfrac{0.85\times3200}{3060}\times100\%\approx89\%$

Q3d $T^n\begin{bmatrix}3200\\2000\\2800\end{bmatrix}$, pick a large $n=100$, $T^{100}\begin{bmatrix}3200\\2000\\2800\end{bmatrix}=\begin{bmatrix}2400\\1600\\4000\end{bmatrix}$

Q3e $R_1=T\begin{bmatrix}3200\\2000\\2800\end{bmatrix}+\begin{bmatrix}200\\100\\k\end{bmatrix}=\begin{bmatrix}3260\\2000\\3040+k\end{bmatrix}$

$R_2=T\begin{bmatrix}3260\\2000\\3040+k\end{bmatrix}+\begin{bmatrix}200\\100\\k\end{bmatrix}=\begin{bmatrix}3333\\2025\\3642\end{bmatrix}$, $k=200$

Q4 Use the information from the table and the incomplete matrix to draw a directed graph.



$$\begin{array}{c} \\ I \\ J \\ K \\ L \\ M \end{array}\begin{array}{c}\begin{array}{ccccc} I & J & K & L & M \end{array}\\ \begin{bmatrix}0&1&1&1&0\\0&0&1&1&1\\0&0&0&1&0\\0&0&0&0&1\\1&0&1&0&0\end{bmatrix}\end{array},\quad \begin{bmatrix}0&1&1&1&0\\0&0&1&1&1\\0&0&0&1&0\\0&0&0&0&1\\1&0&1&0&0\end{bmatrix}^2=\begin{bmatrix}0&0&1&2&2\\1&0&1&1&1\\0&0&0&0&1\\1&0&1&0&0\\0&1&1&2&0\end{bmatrix}$$

Module 2: Networks and decision mathematics

Q1a Missing edge: $A-B$

Q1b Vertex E: degree 2

Q1ci $F-A-B-E-D-C-F$

Q1cii Hamiltonian cycle

Q2a $28+42+16=86$

Q2b Town K

Q3a Minimum cut, maximum flow: $680+650=1330$

Q3bi Vertex A to vertex D

Q3bii Minimum capacity from A to $D=650+130=780$

Q4a $6+5+3=14$ weeks

Q4b 7

Q4c Reduce A by 1 week and L by 2 weeks
Minimum cost (\$) $=1\times140000+2\times120000=380000$

Module 3: Geometry and measurement

Q1a $\frac{4}{3}\pi 2^3 \approx 33.51\,\text{cm}^3$

Q1b $4.1^3 - 33.51 \approx 35.41\,\text{cm}^3$

Q1c $TSA \approx 4.1^2 \times 6 \approx 100.86\ \text{cm}^2$

Q1d $\frac{17.0}{4.1} \approx 4$, $\frac{12.5}{4.1} \approx 3$, $\frac{8.5}{4.1} \approx 2$; max number $= 4 \times 3 \times 2 = 24$

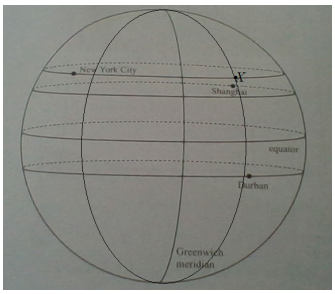
Q2a $A = \frac{1}{2}(2.13 + 4.57) \times 9.75 \approx 32.66\,\text{m}^2$

Q2b $4.57 - 2.13 = 2.44$, $\sqrt{2.44^2 + 9.75^2} \approx 10.0507$
Perimeter $\approx 10.0507 + 4.57 + 9.75 + 2.13 \approx 26.5$ m

Q2c $\overline{AB} = \sqrt{2.7^2 + 3.1^2 - 2(2.7)(3.1)\cos 119°} \approx 5$ m

Q3a Wei-Yi and Ozlem

Q3b



Q3ci Radius $= 6400\cos 41° \approx 4830$ km

Q3cii $74° + 29° = 103°$, distance $= 2\pi \times 4830 \times \frac{103}{360} \approx 8683$ km

Q3d Monday 11 Jan 8 pm Shanghai = Monday 11 Jan 7 am NY
Monday 11 Jan 4 am Durban = Sunday 10 Jan 9 pm NY
Flight time:
Wei-Yi 15 hours; Camilla 25 hours .: 10 hours longer for Camilla

Module 4: Graphs and relations

Q1a 4 minutes

Q1b Average rate $= \frac{200}{2} = 100$ m/minute

Q2a 6 blocks

Q2b 3, 6, 11, 15, 20

Q2c $y \le \frac{3}{5}x$

Q3a $92500 = 10000 + k \times 15$, $k = 5500$

Q3b $6500n = 10000 + 5500n$, $n = 10$

Q3c $P = 6500n - (10000 + 5500n) = 1000n - 10000$
$P = 1000 \times n + (-10000)$

Q4a The total time for x sessions of 30 minutes each for children each day and y sessions of 40 minutes each for adults each day is at most 600 minutes.

Q4b The maximum number of sessions for children each day is 8.

Q4ci $P = 45x + 60y$
Max $P = 45 \times 0 + 60 \times 15 = 900$ when $x = 0$ and $y = 15$

Q4cii $(0, 15)$, $(4, 12)$ and $(8, 9)$

Please inform mathline@itute.com re conceptual and/or mathematical errors