2021 VCAA Mathematical Methods Exam 1 Solutions

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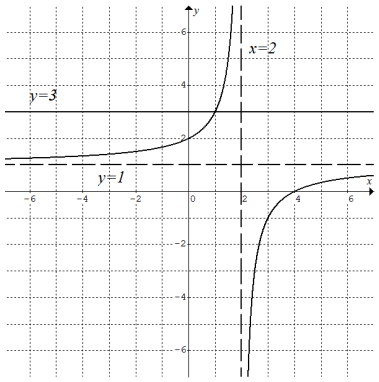
Q1ai $\dfrac{dy}{dx} = -6e^{-3x}$

Q1b $f'(x) = \sqrt{2x+1} + \dfrac{x}{\sqrt{2x+1}}$, $f'(4) = \dfrac{13}{3}$

Q2 $f(x) = \dfrac{x^4}{4} + \dfrac{x^2}{2} + \dfrac{5}{4}$

Q3 $x = \left(\dfrac{1}{6} + n\right)\pi$ or $\left(\dfrac{1}{3} + n\right)\pi$, n is an integer

Q4a and b Solve $1 - \dfrac{2}{x-2} = 3$, $x = 1$ $\therefore$ $x \in [1, 2)$



Q5a $4(x-1)^2 - 4 = 0$, $x = 0, 2$, $(0, 0)$

Q5b

$f(x) = x^2 - 4 \rightarrow f(2x) = (2x)^2 - 4 = 4x^2 - 4$

$\rightarrow f(2(x-2)) = 4(x-2)^2 - 4$

$\therefore$ $h(x) = f(2(x-2)) = 4(x-2)^2 - 4$, intercepts $(1, 0)$, $(3, 0)$

Q6a

	C	C'	
G	0.1	0.2	0.3
G'	0.4	0.3	0.7
	0.5	0.5	1

$\Pr(G' \cap C) = 0.4$

Q6b Total number of glazed donuts $= 0.3 \times 20 = 6$

g glazed donuts in Box A $\therefore$ $0.3 \times 20 - g = 6 - g$ glazed donuts in Box B.

$$\Pr(B \mid \text{glazed}) = \frac{\Pr(B \cap \text{glazed})}{\Pr(\text{glazed})} = \frac{\frac{6-g}{20}}{\frac{6}{20}} = 1 - \frac{g}{6}$$

Q6c $n = 5$, $p = \dfrac{1}{2}$,

$\Pr(\hat{P} \geq 0.8) = \Pr(X \geq 4) = \Pr(X = 4) + \Pr(X = 5)$

$$= \binom{5}{4}\left(\frac{1}{2}\right)^5 + \binom{5}{5}\left(\frac{1}{2}\right)^5 = \frac{3}{16}$$

Q7a $\displaystyle\int_1^2 \frac{k}{x^2}\,dx = 1$, $k = 2$.

Q7b $\mathrm{E}(X) = \displaystyle\int_1^2 x.\frac{2}{x^2}\,dx = 2\log_e 2$

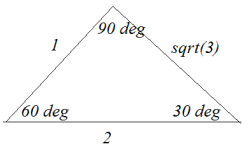
Q8a $\displaystyle y = \int\left(\sqrt{x+6} - \frac{x}{2} - \frac{3}{2}\right)dx = \frac{2(x+6)^{\frac{3}{2}}}{3} - \frac{x^2}{4} - \frac{3x}{2} + C$

Given $\left(3, \dfrac{29}{4}\right)$, $C = -4$ $\therefore$ $y = \dfrac{2(x+6)^{\frac{3}{2}}}{3} - \dfrac{x^2}{4} - \dfrac{3x}{2} + -4$

Q8b Local maximum

x	2	3	4
$\dfrac{dy}{dx}$	positive	0	negative

Q9a



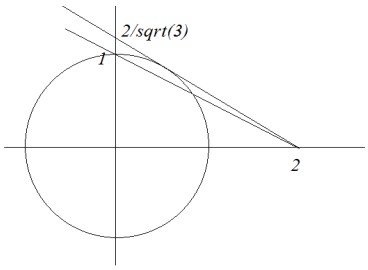
Gradient of line $AP = \tan 150° = -\dfrac{1}{\sqrt{3}}$

Equation of line: $y = -\dfrac{1}{\sqrt{3}}x + c$, Given $(2, 0)$ $\therefore$ $y = -\dfrac{1}{\sqrt{3}}x + \dfrac{2}{\sqrt{3}}$

Q9bi Vertical dilation factor q: $0 < q \leq 1$

Reflection in the x-axis: $-1 \leq q < 0$ $\therefore$ $q \in [-1, 0) \cup (0, 1]$

Q9bii



Two intersections with positive coordinates:

$q < 1$ and $q \times \dfrac{2}{\sqrt{3}} > 1$, $q > \dfrac{\sqrt{3}}{2}$ $\therefore$ $q \in \left(\dfrac{\sqrt{3}}{2}, 1\right)$

Q9ci Area $= \dfrac{1}{2} \times \text{base} \times \text{height} = \dfrac{1}{2} \times 2 \times \sin\theta = \sin\theta$, $\theta \in \left(0, \dfrac{\pi}{3}\right]$

Q9cii P' must be closest to A, maximum area $= \sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$

Please inform mathline@itute.com re conceptual and/or mathematical errors