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2021 VCAA Mathematical Methods Exam 2 Solutions

Use CAS whenever practical

SECTION A – Multiple-choice questions

1	2	3	4	5	6	7	8	9	10
B	C	A	B	C	A	B	E	E	D

11	12	13	14	15	16	17	18	19	20
A	D	B	E	D	A	C	D	E	D

Q1 Period $=\frac{\pi}{\frac{\pi}{2}}=2$ **B**

Q2 $y=\log_e(x)+\log_e(2x)=\log_e(2x^2)$, **C**

Q3 $\text{sd}(\hat{P})=\sqrt{\frac{0.125(1-0.125)}{48}}\approx 0.047735$

$(0.125-1.96\times 0.04774,\ 0.125+1.96\times 0.04774)\approx(0.0314,\ 0.2186)$ **A**

Q4 Graph $h(x)=(x-2)e^x$, $[0,2]$, max $h=0$ at $x=2$ **B**

Q5 $-f(x)=f(-x)$ and $(f(x))^2=f(x^2)$ only **C**

Q6 Binomial: $n=10$, $p=0.25$, $\Pr(X=4)\approx 0.1460$ **A**

Q7 $\frac{dy}{dx}=3x^2-2ax$, at $x=1$, $y=2-a$, $\frac{dy}{dx}=3-2a$

$m=3-2a=\frac{2-a-0}{1-0}$, $a=1$ **B**

Q8 **E**

Q9 $h=f(g(x))=(x+2)^2-4$, graph h to find range **E**

Q10 $x+2\ge 0$ and $1-2x\ge 0$.: $x\ge -2$ and $x\le\frac{1}{2}$ **D**

Q11 $3\int_0^a f(x)\,dx+\int_0^a 2\,dx=3k+2a$ **A**

Q12 $\text{sd}(\hat{P})=\sqrt{\frac{\frac{3}{5}\left(1-\frac{3}{5}\right)}{n}}<0.08$, $n>37.5$, $n=38$ **D**

Q13 Average rate $=\frac{f(12)-f(0)}{12}\approx 10.20$ **B**

Q14 $\int_0^{\pi}\cos\left(kx-\frac{\pi}{2}\right)dx=\int_0^{\pi}\sin x\,dx$, $k=\frac{1}{2}$ **E**

Q15 $\Pr(X=2\mid X\ge 1)=\frac{\Pr(X=2)}{\Pr(X\ge 1)}=\frac{2}{5}$ **D**

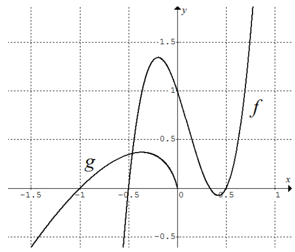
Q16 Fourth quadrant, $\cos x=\frac{3}{5}$, $\sin x=-\frac{4}{5}$;

$\sin^2 y=\frac{25}{169}$, $\cos^2 y=\frac{144}{169}$, $\cos y=\frac{12}{13}$.: $\sin x+\cos y=\frac{8}{65}$ **A**

Q17 $\Pr(X\ge 2)=1-\Pr(X=0)-\Pr(X=1)\ge 0.5$

$1-0.1^0 0.9^n-0.1^1 0.9^{n-1}\ge 0.5$, $n>16.44$, $n=17$ **C**

Q18 Translate graph f to the left to show a maximum of 3 intersections. **D**



Q19 Continuous and smooth **E**

Q20 A and B are independent .: $\Pr(A\cap B)=\Pr(A)\Pr(B)=p^3$

.: $\Pr(A'\cap B)=p^2-p^3$, $\Pr(A'\cup B)=\Pr(A')+\Pr(B)-\Pr(A'\cap B)$

$=(1-p)+p^2-(p^2-p^3)=1-p+p^3$ **D**

SECTION B

Q1a $V_{box}=height\times width\times length=x(h-2x)(2h-2x)$

$=x(25-2x)(50-2x)=2x(25-2x)(25-x)$

Q1b $V_{box}>0$, $x>0$, $25-2x>0$ i.e. $x<\frac{25}{2}$.: domain is $\left(0,\frac{25}{2}\right)$

Q1c $(V_{box})'=2(6x^2-150x+625)$

Q1d Let $(V_{box})'=2(6x^2-150x+625)=0$, max volume when

$x=\frac{25(3-\sqrt{3})}{6}$, max $V_{box}=\frac{15625\sqrt{3}}{9}$ cm^3

Q1e When $x=5$, % wasted $=\frac{4\times 5^2}{25\times 50}=0.08=8\%$

Q1fi $V_{box}=x(h-2x)(2h-2x)$, $h-2x>0$, $x<\frac{h}{2}$.: domain $\left(0,\frac{h}{2}\right)$

Q1fii Let $(V_{box})'=0$, $12x^2-12hx+2h^2=0$, $x=\frac{(3-\sqrt{3})h}{6}\in\left(0,\frac{h}{2}\right)$

Max $V_{box}=\frac{\sqrt{3}\,h^3}{9}$

Q1g $V=x(h-2x)^2>0$, $h-2x>0$, .: domain $\left(0,\frac{h}{2}\right)$

Let $V'=(h-2x)(h-6x)=0$.: $h-6x=0$, $x=\frac{h}{6}$

When $x<\frac{h}{6}$, $V'>0$; when $x>\frac{h}{6}$, $V'<0$.: max V at $x=\frac{h}{6}$

Q2a $\frac{1}{4}$

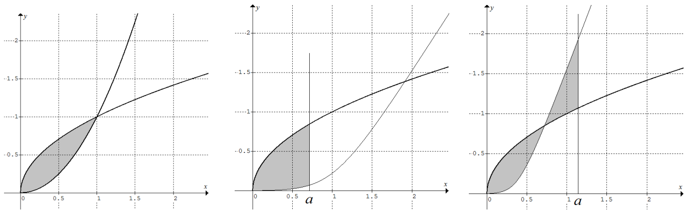
Q2b Total area $=\frac{15}{32}$

Q2c $\int_0^1 x^2\,dx=\frac{1}{3}$

Q2d Approxation $\approx 1\times(6+2-4-6)=-2$

Q2e Area of shaded region $= 2\left(\frac{1}{2}\times1\times1-\frac{1}{3}\right)=\frac{1}{3}$

Q2f Three possibilities: $a=1$, $0<a<1$ and $1<a\le 2$



When $0<a<1$, $\int_0^a\left(\sqrt{x}-ax^2\right)dx=\frac{1}{3}$, $a\approx 0.77$

When $1<a\le 2$: intersection $\sqrt{x}=ax^2$, $x=a^{-\frac{2}{3}}$

$\int_0^{a^{-\frac{2}{3}}}\left(\sqrt{x}-ax^2\right)dx-\int_{a^{-\frac{2}{3}}}^{a}\left(ax^2-\sqrt{x}\right)dx=\frac{1}{3}$, $a\approx 1.13$

Q3a $x^2-1>0$ AND $1-x>0$.: $x<-1$, max domain is $(-\infty,-1)$
Range is R.

Q3bi $q=0$ and $\frac{dq}{dx}=\frac{2x}{x^2-1}+\frac{1}{1-x}=-1$ at $x=-2$

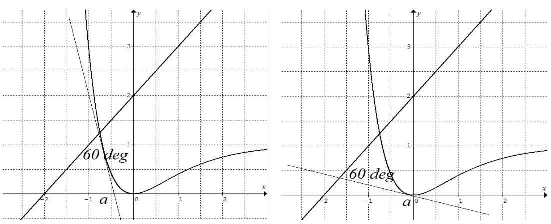
.: equation of tangent: $\frac{y-0}{x-(-2)}=-1$, $y=-x-2$

Q3bii Gradient of normal is 1. Normal: $\frac{y-0}{x-(-2)}=1$, $y=x+2$

Q3c $p(x)=e^{-2x}-2e^{-x}+1$, $p'(x)=-2e^{-2x}+2e^{-x}=0$ when $x=0$
It has a turning point at $(0,0)$, consider the line $y=0.5$, it cuts the curve of $p(x)$ at two places .: not a one-to-one function

Q3d $p'(a)=-2e^{-2a}+2e^{-a}$

Q3e Two possibilities:



Let $\alpha°$ be the angle between the tangent and the positive x-axis.
$\alpha=45+60=105$ or $\alpha=45+120=165$
$p'(a)=-2e^{-2a}+2e^{-a}=\tan105°$ or $\tan165°$
.: $\alpha\approx-0.67$ or -0.11

Q3f Intersection: $x+2=e^{-2x}-2e^{-x}+1$, $x\approx-0.750$

Area $\approx\int_{-2}^{-0.750}(x+2)dx+\int_{-0.750}^{0}\left(e^{-2x}-2e^{-x}+1\right)dx\approx 1.038$

Q4a $\Pr(W\ge 11)\approx 0.106$

Q4b $\Pr(W<k)=0.8$, $\frac{k-10}{0.8}\approx 0.84162$, $k\approx 10.7$

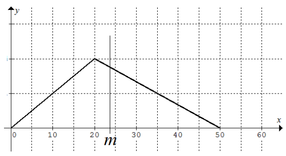
Q4c $E(\hat{P})=p=0.08$, $\text{sd}(\hat{P})=\sqrt{\frac{0.08\times0.92}{25}}=\frac{\sqrt{46}}{125}$

Q4d $\Pr(\hat{P}>0.1)=\Pr(X>0.1\times25)=1-\Pr(X\le 2)\approx 0.323$

Q4e $x_{\max}=50$

Q4f Let m be the median.

$\frac{1}{2}\times\frac{50-m}{750}\times(50-m)=0.5$
$m\approx 22.6$



Q4g $E(X)=\int_0^{20}\frac{x^2}{500}dx+\int_{20}^{50}\frac{x(50-x)}{750}=\frac{70}{3}$,

$\text{Var}(X)=E(X^2)-(E(X))^2=\int_0^{20}\frac{x^3}{500}dx+\int_{20}^{50}\frac{x^2(50-x)}{750}-\left(\frac{70}{3}\right)^2$

≈ 105.547 .: $\text{sd}(X)=\sqrt{\text{Var}(X)}\approx 10.3$

Q4h $g(x)=af\left(\frac{x}{b}\right)=\frac{a\left(50-\frac{x}{b}\right)}{750}$ for $20b\le x\le 50b$ and $20b\le m\le 50b$
where m is the median.

Area under the prob density function $=\frac{1}{2}\times50b\times\frac{a}{25}=1$.: $ab=1$

Given $m=30$.: $\frac{1}{2}(50b-30)\times\frac{a\left(50-\frac{30}{b}\right)}{750}=0.5$.: $a\approx 0.75$, $b\approx 1.33$

Q5a Period $=4\pi$

Q5b Min of $f\approx-1.722$

Q5c $h=2\pi$

Q5d $a=2$

Q5ei $\int g_a\,dx=-a\cos\left(\frac{x}{a}\right)+\frac{\sin(ax)}{a}$

Q5eii $\int_0^{2a\pi}g_a\,dx=\left[-a\cos\left(\frac{x}{a}\right)+\frac{\sin(ax)}{a}\right]_0^{2a\pi}=0$

.: equal area bounded by g_a and the x-axis over the interval $[0,2a\pi]$ above and below the x-axis for all values of a

Q5f $-1\le\sin\left(\frac{x}{a}\right)\le 1$, $-1\le\cos(ax)\le 1$.: $-2\le g_a\le 2$ for all a

Q5g $a=1,2,3,\ldots$ When $a=1$, min $g_a=-\sqrt{2}$
As the value of a increases, the minimum point of g_a is lower
.: the greatest minimum value of g_a is $-\sqrt{2}$

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