2022 Mathematical Methods Trial Exam 1 Solutions © 2022 itute

Question 1

Second equation: $2x-z-3=0$, $x=\frac{3+z}{2}$ Let $z=\alpha$.: $x=\frac{3+\alpha}{2}$

First equation: $x-\frac{3}{2}y+z=0$, $2x-3y+2z=0$, $y=\frac{2x+2z}{3}$, $y=1+\alpha$

Alternatively:

Second equation: $2x-z-3=0$, $z=2x-3$ Let $x=\alpha$.: $z=2\alpha-3$

First equation: $x-\frac{3}{2}y+z=0$, $2x-3y+2z=0$, $y=\frac{2x+2z}{3}$, $y=2\alpha-2$

Question 2

a.

$$\begin{array}{r} 3x^3-\frac{3}{2}x^2+x-\frac{1}{2} \\ 2x+1\,)\;6x^4+0x^3+\frac{1}{2}x^2+0x+1 \\ -\left(6x^4+3x^3\right) \\ \hline -3x^3+\frac{1}{2}x^2 \\ -\left(-3x^3-\frac{3}{2}x^2\right) \\ \hline 2x^2+0x \\ -\left(2x^2+x\right) \\ \hline -x+1 \\ -\left(-x-\frac{1}{2}\right) \\ \hline \frac{3}{2} \end{array}$$

.: $Q(x)=3x^3-\frac{3}{2}x^2+x-\frac{1}{2}$ and $R=\frac{3}{2}$

b. Remainder $=\left(2\left(\frac{1}{2}\right)+1\right)Q\left(\frac{1}{2}\right)=0$

Question 3

a. $y=a\log_e(bx-c)=a\log_e b\left(x-\frac{c}{b}\right)$

Asymptote, $x=\frac{c}{b}=1.5$; $(2,0)$, $a\log_e b(2-1.5)=0$.: $\log_e\frac{b}{2}=0$.: $b=2$ and $c=3$

b. $y=a\log_e(2x-3)$ Let $2x-3=e\approx 2.7$.: $x\approx 2.85$ and from graph $y\approx 3$ $y=a\log_e e$.: $a=y\approx 3$

Question 4

A possible sequence:

(1) Vertical dilation by a factor of ½. (2) Horizontal dilation by a factor of ½. (3) Reflection in the *y*-axis.

(4) Translation to the right by ½ units.

Question 5

a. Require range of $g(x)\subseteq$ domain of $f(x)$. Since the domain of $f(x)$ is $[1,\infty)$

.: the lowest value of $g(x)=4x-x^2$ must be 1, let $1=4x-x^2$, $x^2-4x+1=0$, $x=2-\sqrt{3}$ or $2+\sqrt{3}$

The highest value of $g(x)=4x-x^2$ is 4 when $x=2$, and $4\in[1,\infty)$

.: minimum $p=2-\sqrt{3}$ and maximum $q=2+\sqrt{3}$

b. $y=f(g(x))=\sqrt{4x-x^2+1}+1$, $\frac{dy}{dx}=\frac{4-2x}{2\sqrt{4x-x^2+1}}=\frac{2-x}{\sqrt{4x-x^2+1}}=-\frac{1}{2}$

Squaring both sides: $\frac{4-4x+x^2}{4x-x^2+1}=\frac{1}{4}$.: $x=1$ or 3 Only at $x=3$, $\frac{dy}{dx}=-\frac{1}{2}$

Question 6

a. Both T_A and T_B have the same period 24 hours $\therefore T_A \geq T_B$ for a half of a period, i.e. 12 hours

b. $T_A = T_B$, $\cos\left(\frac{\pi}{12}t\right) = -\sin\left(\frac{\pi}{12}t\right)$ $\therefore \tan\left(\frac{\pi}{12}t\right) = -1$, $t = 9, 21, \ldots$

At $t = 21$, $\frac{dT_A}{dt} = -\frac{\pi}{6}\sin\left(\frac{\pi}{12}t\right) > 0$ and $\frac{dT_B}{dt} = -\frac{\pi}{6}\cos\left(\frac{\pi}{12}t\right) < 0$

Question 7

a. $\Pr(X<2) = \Pr(X=0) + \Pr(X=1) = \left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^n + \left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^{n-1} > \frac{20}{27}$

When $n=1$, $\Pr(X<2)=1$; when $n=2$, $\Pr(X<2)=\frac{8}{9}$; when $n=3$, $\Pr(X<2)=\frac{20}{27}$

$\Pr(X<2)$ decreases as n increases $\therefore n=2$ is the greatest value

b. For $\text{Bi}\left(33, \frac{1}{3}\right)$, $\bar{X} = np = 33 \times \frac{1}{3} = 11$

Question 8

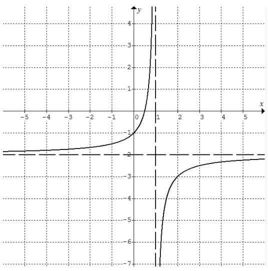
a. $\int_0^e ke^{\left(\frac{x}{e}+1\right)}dx = 1$, $\int_0^e kee^{\left(\frac{x}{e}\right)}dx = 1$, $\left[ke^2 e^{\left(\frac{x}{e}\right)}\right]_0^e = 1$, $ke^3 - ke^2 = 1$ $\therefore k = \frac{1}{e^2(e-1)}$

b. Let the median be m. $\int_0^m \frac{e}{e^2(e-1)}e^{\left(\frac{x}{e}\right)}dx = \frac{1}{2}$, $\left[\frac{1}{e-1}e^{\left(\frac{x}{e}\right)}\right]_0^m = \frac{1}{2}$

$\frac{1}{e-1}\left(e^{\left(\frac{m}{e}\right)} - 1\right) = \frac{1}{2}$, $e^{\left(\frac{m}{e}\right)} = \frac{e+1}{2}$, $\frac{m}{e} = \log_e\left(\frac{e+1}{2}\right)$ $\therefore m = e\log_e\left(\frac{e+1}{2}\right)$

Question 9

a. $\frac{a}{x-1} + b = \frac{2x-1}{1-x}$, $\frac{a+bx-b}{x-1} = \frac{1-2x}{x-1}$ $\therefore b=-2$ and $a=-1$



Asymptotes: $x=1$, $y=-2$

b. $x = \frac{2y-1}{1-y}$, $x - xy = 2y - 1$, $x+1 = (x+2)y$, $y = \frac{x+1}{x+2}$, $f^{-1}(x) = \frac{x+1}{x+2}$

c. $x = \frac{2x-1}{1-x}$, $x^2 + x - 1 = 0$, $x = \frac{-1-\sqrt{5}}{2}$, $x = \frac{-1+\sqrt{5}}{2}$ Coordinates: $\left(\frac{-1-\sqrt{5}}{2}, \frac{-1-\sqrt{5}}{2}\right)$, $\left(\frac{-1+\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right)$

d. $\text{Area} = 2 \times \int_{\frac{-1-\sqrt{5}}{2}}^{\frac{-1+\sqrt{5}}{2}} \left(x - \frac{2x-1}{1-x}\right)dx = 2 \times \int_{\frac{-1-\sqrt{5}}{2}}^{\frac{-1+\sqrt{5}}{2}} \left(x + 2 - \frac{1}{1-x}\right)dx = \left[(x+2)^2 + 2\log_e(1-x)\right]_{\frac{-1-\sqrt{5}}{2}}^{\frac{-1+\sqrt{5}}{2}}$

$$= \left(\frac{-1+\sqrt{5}}{2}+2\right)^2 + 2\log_e\left(1 - \frac{-1+\sqrt{5}}{2}\right) - \left(\frac{-1-\sqrt{5}}{2}+2\right)^2 - 2\log_e\left(1 - \frac{-1-\sqrt{5}}{2}\right)$$

$$= \left(\frac{3+\sqrt{5}}{2}\right)^2 + 2\log_e\left(\frac{3-\sqrt{5}}{2}\right) - \left(\frac{3-\sqrt{5}}{2}\right)^2 - 2\log_e\left(\frac{3+\sqrt{5}}{2}\right) = 3\sqrt{5} + 2\log_e\left(\frac{3-\sqrt{5}}{3+\sqrt{5}}\right)$$

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