itute.com

Online & home tutors Registered business name: itute ABN: 96 297 924 083

Mathematical Methods

2022

Trial Examination 1
(1 hour)

Instructions

Answer **all** questions.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1 (3 marks)
Solve the following simultaneous equations for x, y and z in terms of a parameter.
Let the parameter be α.

$$x-\frac{3}{2}y+z=0$$
$$2x-z-3=0$$

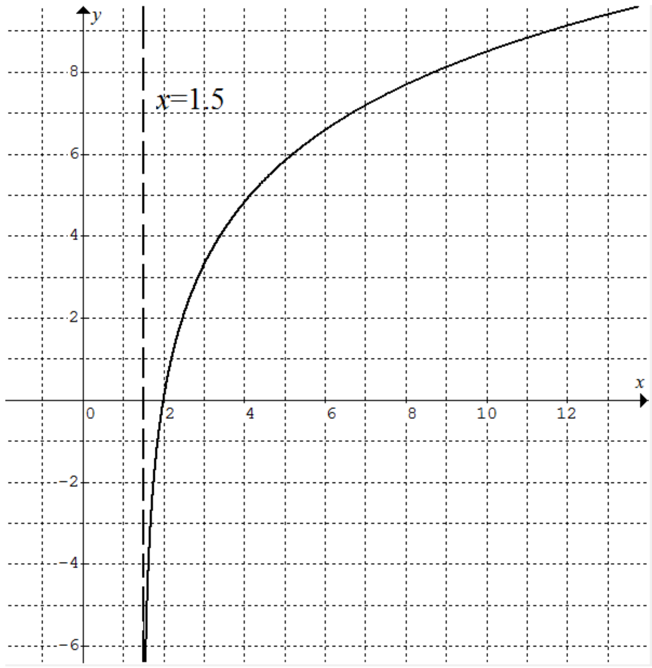
Question 2 (4 marks)
Given $P(x)=6x^4+\frac{1}{2}x^2+1=(2x+1)Q(x)+R$ where $Q(x)$ is a polynomial function and R is a real constant.

a. Find $Q(x)$ and R. 3 marks

b. Find the remainder when $(2x+1)Q(x)$ is divided by $2x-1$. 1 mark

Question 3 (4 marks)

The graph of a logarithmic function of equation in the form $y = a\log_e(bx - c)$ is shown below. It has an asymptote $x = 1.5$ and intercept $(2,0)$.



a. Show that $b = 2$ and $c = 3$.

2 marks

b. Given $e \approx 2.7$.

Use the graph to determine the approximate value of a corrected to the nearest whole number.

2 marks

Question 4 (4 marks)
Function f has equation $y = f(x)$.

State a sequence of four transformations of function f, changing $y = f(x)$ to $y = \frac{1}{2} f(1-2x)$.

Question 5 (5 marks)
Consider $f(x) = \sqrt{x+1} + 1,\ x \in [1, \infty)$ and $g(x) = 4x - x^2,\ x \in [p, q]$.

a. Find the minimum value of p and the maximum value of q such that composite function $f(g(x))$ is defined.

3 marks

b. Find x such that the gradient of the tangent to $y = f(g(x))$ is $-\frac{1}{2}$. 2 marks

Question 6 (4 marks)

The temperatures of Room A and Room B at time t (hours) are given by $T_{\text{A}} = 25 + 2\cos\left(\frac{\pi}{12}t\right)$ and $T_{\text{B}} = 25 - 2\sin\left(\frac{\pi}{12}t\right)$ respectively for $t \geq 0$.

a. For how long is $T_{\text{A}} \geq T_{\text{B}}$ in the first 24 hours? 1 mark

b. Find t such that $T_{\text{A}} = T_{\text{B}}$, $\frac{dT_{\text{A}}}{dt} > 0$ and $\frac{dT_{\text{B}}}{dt} < 0$ for the first time. 3 marks

Question 7 (4 marks)

Random variable X has a binomial probability distribution given by $\text{Bi}\left(n, \frac{1}{3}\right)$.

a. Find the greatest value of n such that $\Pr(X < 2) > \frac{20}{27}$. 2 marks

b. Find $\overline{X}$ when $n = 33$. 1 mark

Question 8 (4 marks)

Random variable X has probability density function $f(x)$ given by

$$f(x)=\begin{cases} ke^{\left(\frac{x}{e}+1\right)} & \text{for } 0 \le x \le e \\ 0 & \text{elsewhere} \end{cases}$$

a. Show that $k=\dfrac{1}{e-1}$. 2 marks

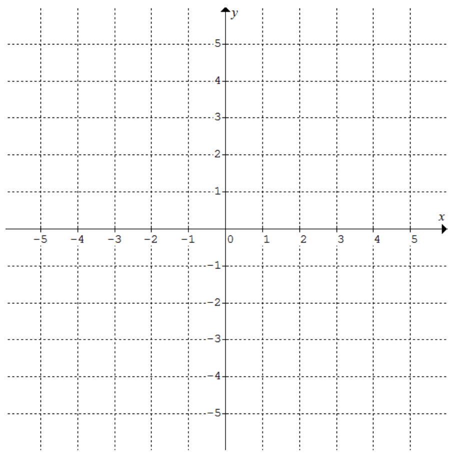
b. Find the median of X. 2 marks

Question 9 (8 marks)

Let $f(x)=\dfrac{2x-1}{1-x}$.

a. $f(x)$ can be expressed as in the form $\dfrac{a}{x-1}+b$.

Find the value of each of a and b. Hence sketch the graph of $y=f(x)$. 3 marks



b. Determine $f^{-1}(x)$. 1 mark

c. Find the coordinates of the intersections of $y=f(x)$ and $y=f^{-1}(x)$. 1 mark

d. Find the area of the region enclosed by $y=f(x)$ and $y=f^{-1}(x)$.
Express your answer in simplest form. 3 marks

End of Exam

