itute.com

Online & home tutors *Registered business name: itute ABN: 96 297 924 083*

2022 Mathematical Methods

Year 12

Problem Solving Task

Time allowed: 2 hours plus

Problem Solving Task

Theme: NAPLAN tests

The National Assessment Program—Literacy and Numeracy (NAPLAN) tests are conducted in May for all students across Australia in Years 3, 5, 7 and 9. Each year, over one million students nationally sit the NAPLAN tests. All students in the same year level are assessed on the same test items in the assessment domains of reading, writing and numeracy.

Assumed knowledge:

Discrete and continuous random variables; probability mass functions and density functions; binomial distributions, normal distributions and approximation; conditional probability; sample proportion and statistical inference; calculus; CAS

Part I (60 minutes plus)

A year 11 class has twelve students. The proportion of girls in the class is $\frac{2}{3}$.

Three students are selected to organize the NAPLAN tests for their school.

a. Draw a tree diagram with probability values to show all possible selections (boy or girl) of the first, second and third students.

b. Calculate the probability that the selection has two girls and a boy.

The year 11 level at the school has 120 students and the proportion of girls is also $\frac{2}{3}$.

c. Instead of following the steps in part a and part b, use an appropriate probability distribution to approximate the probability that the selection has two girls and a boy. Justify the use of your approximation.

Random variable score X (in whole number) out of 100 of a particular NAPLAN test for 2500 students has probability mass function $p(x)$ such that

$$p(55)+p(56)+p(57)+\cdots\cdots+p(64)+p(65)=0.68$$

d. Find the number of students scoring $55 \le x \le 65$.

e. The probability mass function $p(x)$ can be approximated with a normal probability density function $f(x)$.
Use this approximation to estimate $\Pr(63 \le X < 73)$ and the number of students scoring $63 \le x < 73$.

f. Estimate $\Pr(63 \le X < 73)$ if $55 \le x \le 65$.

g. Let event A be $55 \le X \le 65$ and event B be $63 \le X < 73$.
Find $\Pr(B' \mid A)$ and $\Pr(B' \mid A')$.

Score X (in whole number) out of 100 of another NAPLAN test nationally has a normal distribution with probability density function $f(x) = ae^{-0.0319x^2 + 3.6990x - 107.27}$.

h. Find the value of a in $f(x)$.

i. Sketch the graph of $f(x)$ for $x \in [0, 100]$ and determine the mean score μ of the test.

j. Determine the 95% confidence interval of X, and hence the variance of X, $\text{Var}(X)$.

k. Use an alternative method to find $\text{Var}(X)$.

End of Part I

Part II (60 minutes plus)

The National Assessment Program—Literacy and Numeracy (NAPLAN) tests are conducted in May for all students across Australia in Years 3, 5, 7 and 9. Each year, over one million students nationally sit the NAPLAN tests. All students in the same year level are assessed on the same test items in the assessment domains of reading, writing and numeracy.

In a particular year, numeracy test for Year 9 students across Australia has a mean score of 583.6 (out of 1000) and a standard deviation of 82.2. Across Victoria the mean is 588.4, and the standard deviation is 77.9.

Assume that the Year 9 numeracy test scores in Victoria and nationally are normally distributed in the particular year.

a. Determine the percentage of **Victorian** Year 9 students above the **national** mean in the numeracy test.

b. Determine the lowest and the highest numeracy test scores (correct to the nearest unit) of the middle 68% of Year 9 students **in Victoria**.

c. Determine the percentage of Year 9 students **nationally** scored within 588.4 ± 77.9 in the numeracy test.

d. Find the probability that there are more than 6 among 10 randomly chosen **Victorian** Year 9 students scoring above the **national** mean in the numeracy test.

e. Determine the percentage scored below 788 among the **Victorian** Year 9 students scoring above the **national** mean in the particular year.

f. In the numeracy test, 80% of **NSW** Year 9 students scored below 668 and 70% scored above 544. Calculate the mean and standard deviation of the NSW numeracy test scores assuming a normal distribution of the scores.

g. Sarah did the Year 9 numeracy test in **Victoria** and she was in the top 5% in Victoria. If she did the same test in **NSW** she would be in the top $x\%$ in NSW.
Find x correct to the nearest whole number.

All students across Australia do the NAPLAN tests in Years 3, 5, 7 and 9. Suppose the probability that a student scores above the national mean in a test is 0.90 if the student scores above the national mean in the previous test; and the probability that the student scores below the national mean in a test is 0.60 if the student scores below the national mean in the previous test.

h. If a student scores below the national mean in the Year 3 test, find the probability that the student will score above the national mean in the next three tests.

i. If the student scored above the national mean in the Year 3 and Year 5 tests, find the probability that the student will score above the national mean in at least one of the next two tests.

j. If the student scores above the national mean in the Year 3 and Year 7 tests, find the expected number of times scoring above the national mean in the four tests.

Repeat statement: In a particular year, numeracy test for Year 9 students across Australia has a mean score of 583.6 (out of 1000) and a standard deviation of 82.2.
Assume that the test scores nationally are normally distributed.

25 random samples of size 100 (year 9 student test results) are taken from that year.

k. Determine the mean of the proportions of students scoring higher than 650 in a sample.

l. Determine the standard deviation of the proportions of students scoring higher than 650 in a sample.

m. How many (to nearest unit) of the random samples have the proportion of students scoring higher than 650 greater than 0.2?

n. Find the size of a sample if the standard deviation of the proportions of students scoring higher than 650 in a sample is halved.

Another random sample of size $n = 2500$ (year 9 student test results) is taken from that year. The proportions of students in the sample scoring higher than 650 is $\hat{p} = 0.2102$.

o. Calculate the interval $\left(\hat{p} - 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$.

p. Does this interval contain the actual national proportion for that year?

q. Consider taking 20 such random samples.
$\hat{p}$ and interval $\left(\hat{p} - 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$ are calculated for each sample.
How many intervals NOT containing the actual national proportion for that year do you expect to find?

r. Explain whether or not the interval $\left(\hat{p} - 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$ in **part o** is a good estimator of the proportion of year 9 students nationally scoring higher than 650 in future years.

End of Part II