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Specialist Mathematics

2022

Trial Examination 1
(1 hour)

Instructions

Answer **all** questions. Do **not** use calculators.
Unless otherwise specified, an **exact** answer is required to a question.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
In questions where more than one mark is available, show appropriate working or explanation.
Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$

Question 1 (5 marks)
The distribution of the height X of the Victorian year 12 students in a random sample of 100 is approximately normal.

a. Given that $\Pr(X < 154.4) \approx 0.025$ and $\Pr(X > 193.6) \approx 0.025$, show that the mean and standard deviation of the heights (cm) of students in the sample are 174 and 10 respectively.

2 marks

b. 60 students in the sample has a mean height of 170 cm. Find the mean height of the remaining students in the random sample.

1 mark

c. Find an approximate 95% confident interval of the mean height of the Victorian year 12 students. 2 marks

Question 2 (4 marks)

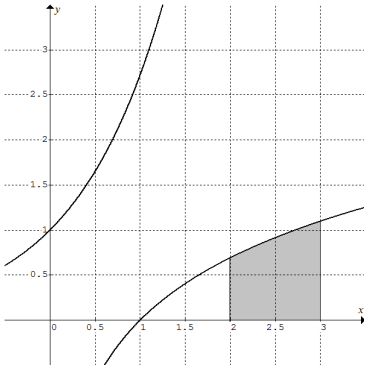
Solve the following equations for x over C.

a. $x^4 + 3x^2 - 4 = 0$. 2 marks

b. $x^4 + 3x^2 + 4 = 0$. 2 marks

Question 3 (4 marks)

The graphs of $y = \log_e x$ and $y = e^x$ are shown below. The area of the shaded region is given by $\int_2^3 \log_e x \, dx$.

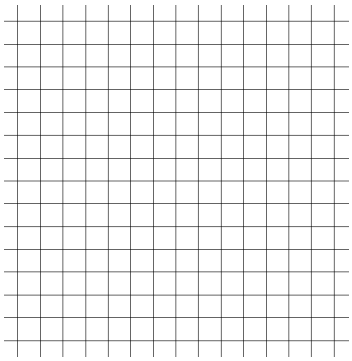


a. Show that $\int_0^{\log_e 3} e^x \, dx - \int_0^{\log_e 2} e^x \, dx = 1$. 1 mark

b. Hence evaluate $\int_2^3 \log_e x \, dx$. 3 marks

Question 4 (3 marks)

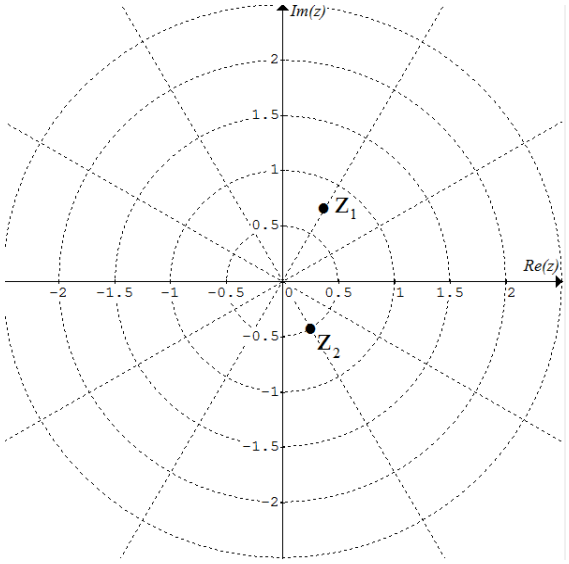
a. On the grid diagram below sketch the graph of $f(x) = \cos^{-1}|x|$. Show axis intercepts. 2 marks



b. Find the area of the region bounded by $f(x) = \cos^{-1}|x|$ and the x-axis. 1 mark

Question 5 (4 marks)

Two complex numbers, z_1 and z_2, are shown in the following diagram.



Mark accurately (correct modulus and argument) with a dot and label each one of the following four complex numbers on the above diagram.

a. $z_3 = -i\,z_1$ 1 mark

b. $z_4 = z_1 - z_2$ 1 mark

c. $z_5 = 4z_1\,\overline{z}_2$ 1 mark

d. $z_6 = \dfrac{-z_1}{z_2}$ 1 mark

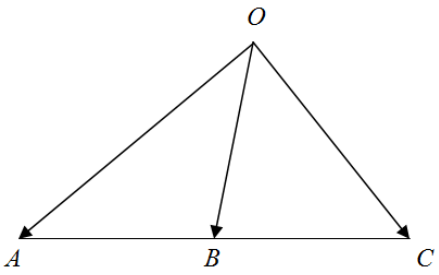
Question 6 (5 marks)

a. Given $\displaystyle\int_0^{\pi} \sin^{-1}\left(\frac{x}{\pi}\right)dx = \frac{\pi}{2}(\pi - 2)$, and $(0, 2\pi)$ is a point on the solution curve of differential equation

$\dfrac{dy}{dx} = 2\sin^{-1}\left(\dfrac{x}{\pi}\right)$, find y when $x = \pi$. 2 marks

b. Find $\frac{d}{dx}\left(\frac{x}{\pi}\sin^{-1}\left(\frac{x}{\pi}\right)+\sqrt{1-\left(\frac{x}{\pi}\right)^2}\right)$ and express your answer in simplest form. 3 marks

Question 7 (5 marks)

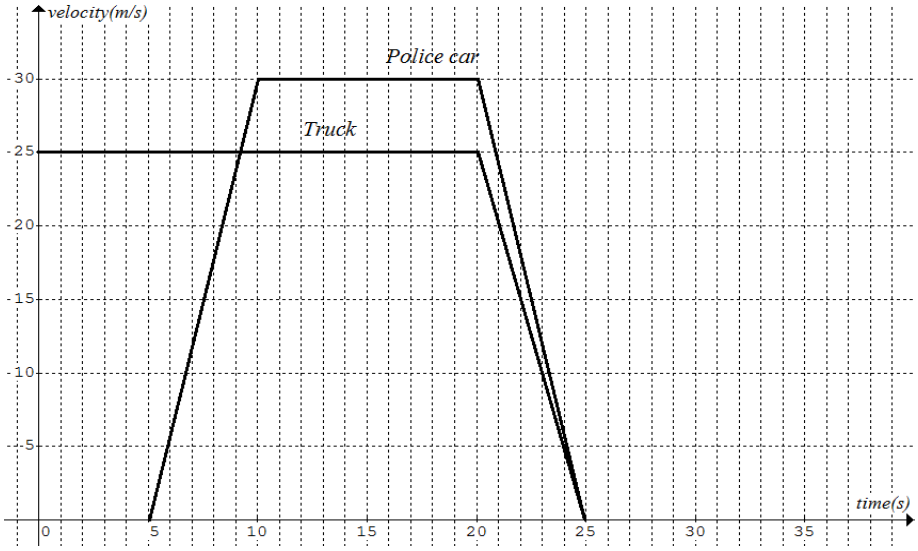


The diagram above shows three position vectors $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OC}$. Points A, B and C are collinear, $\frac{\overrightarrow{OA}.\overrightarrow{OB}}{\left|\overrightarrow{OB}\right|}=\frac{\overrightarrow{OC}.\overrightarrow{OB}}{\left|\overrightarrow{OB}\right|}$ and Q is a point on AC such that $AQ=QC=OQ$.

a. Show that line segments OB and AC are perpendicular. 2 marks

b. Show that $\overrightarrow{OA}$ and $\overrightarrow{OC}$ are perpendicular. 3 marks

Question 8 (5 marks)



The unrealistic velocity-time graphs of a police car and a truck along a straight road are shown above.
Velocity is in metres per second, and time t is in seconds.
The truck passes a post office at $t = 0$ at constant velocity 25 m s^{-1}, and it passes the stationary police car at $t = 5$. The police starts to chase at $t = 5$. The maximum speed of the police car is 30 m s^{-1}.
Read the graphs for other relevant information.

a. What is the distance of the police car (m) from the post office at $t = 5$. 1 mark

b. Find the distance the police car is ahead of the truck when both stop at $t = 25$. 1 mark

c. Write a piecewise function $D(t)$ giving the distance travelled by the police car at time t in the time interval $[5, 25]$.

3 marks

Question 9 (5 marks)

Two forces (in newtons), $\vec{F}_1 = 20\ \text{N}\,30°\,\text{E}$ and $\vec{F}_2 = 15\ \text{S}\,60°\,\text{E}$, act on a 2-kg particle, and there are no other forces. The particle is initially at rest.

a. Find the magnitude of the resultant force. 1 mark

b. The direction of the resultant force can be expressed in the form $\text{N}\ \left(\sin^{-1}(a) + \sin^{-1}(b)\right)°\ \text{E}$.
Find the values of a and b where $a < b$. 2 marks

c. Find the average speed of the particle in the first 5 seconds. 2 marks

End of Exam 1