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2022 Specialist Mathematics

Year 11 Application Task

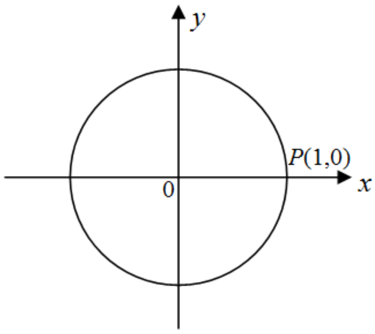
(Time allowed: 2 hours plus)

Theme: Falling behind

Assumed knowledge: Circle; circumference; uniform speed; distance as a function of time t; highest common factor (HCF); lowest common multiple (LCM); coordinates; $x(t)$, $y(t)$ and graphs; CAS

Introduction and specifications:

Two particles, A and B, travel continuously at constant speed around the same circular path of radius 1 unit. Particle A completes 1 round in a seconds, and Particle B completes 1 round in b seconds where $b < a$, $a, b \in Z^{+}$.
Both particles start from point $P(1, 0)$ at time $t = 0$ in the anticlockwise direction. t is in seconds.



Part I (75 minutes plus)

a. Express the circumference of the circle in terms of π.

b. Find the speeds (in units per second) of Particle A and Particle B in terms of π, a and b.

c. Find the distance travelled by each particle at time t.

d. Which particle has fallen behind?

e. Show that at time $t \le b$, one particle is behind the other particle by $\frac{2(a-b)\pi t}{ab}$ units.

f. The first time the particles are together is when $t = 0$.

Show that the particles are together for the second time when $t = \frac{ab}{a-b}$.

g. Find t in terms of a and b when the particles are together for the third time and the fourth time.

Consider $a = 24$ and $b = 22$.

h. Find the values of t when the particles are together for the second time, the third time and the fourth time.

i. Find the coordinates of the particles when they are together for the second time, the third time and the fourth time.

Now consider $a = 39$ and $b = 30$.

j. Find the values of t and the coordinates of the particles when they are together for the second time, the third time and the fourth time.

k. Compare and comment on the findings in parts h, i and j.

l. Case 1: Suppose $a = k(n+1)$ and $b = kn$ where $k, n \in Z^+$ and k is the HCF of a and b.
Find the values of t in terms of k and n, and the coordinates of the particles when they are together for the second time, the third time and the fourth time.

m. Case 2: Suppose $a = k(n+m)$ and $b = kn$ where $k, n, m \in Z^+$, k is the HCF of a and b, and $m > 1$.
In terms of k, n and m, find the value of t, and the coordinates of the particles when they are together for the second time.

n. In each of Case 1 and Case 2, write down the LCM of a and b.
Hence, in terms of one/both of LCM and m, find the value of t, and the coordinates of the particles when they are together for the second time in each case.

o. Select an appropriate value (greater than 39) for each of a and b satisfying the requirements in **Case 2**.
Use the results in part n and your selected a and b values to find the value of t, and the coordinates of the particles when they are together for the second time.

End of Part I

Introduction and specifications from Part I:

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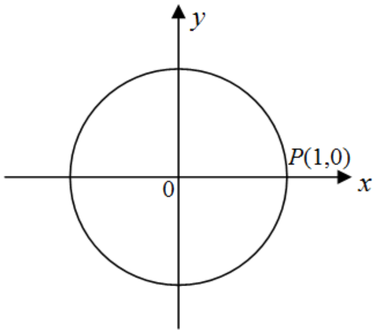
Both particles start from point $P(1, 0)$ at time $t = 0$ in the anticlockwise direction. t is in seconds.



Part II (55 minutes plus)

At time t, the y-coordinate $y_A(t)$ of Particle A is given by $y_A(t) = \sin \eta t$

a. Show that $\eta = \dfrac{2\pi}{a}$ and write $y_A(t)$ in terms of a.

b. Write down the y-coordinate $y_B(t)$ of Particle B in terms of b.

c. Express the x-coordinates, $x_A(t)$ and $x_B(t)$, of Particle A and Particle B respectively in terms of a and b.

d. Show that Particle A and Particle B share the same path.

Let $a = 42$ and $b = 24$.

e. Use your CAS to display the graph of $y_A(t)$ and $y_B(t)$ for $a = 42$ and $b = 24$ over the interval $t \in [0, 180]$. Do not sketch the graphs.
Identify the values of t in the interval when Particle A and Particle B are together momentarily.

f. Determine the exact coordinates of the particles when they are together momentarily.

Use the identity $\sin X - \sin Y = 2\sin\frac{X-Y}{2}\cos\frac{X+Y}{2}$ to answer part g to part

g. Express $y_B(t) - y_A(t)$, i.e. $\sin\frac{2\pi t}{24} - \sin\frac{2\pi t}{42}$ as the product of sine and cosine functions.

h. Hence the values of $t \in [0, 180]$ when Particle A and Particle B are together momentarily.
Without referring to part e, explain how these values (but not the others) are chosen.

i. Find the values of $t \in [180, 360]$ when Particle A and Particle B are together momentarily.

End of Part II   End of Task

