2022 VCAA Mathematical Methods Exam 1 Solutions

Q1a Apply the product rule and factorise: $\frac{dy}{dx}=e^{2x}(6x+3)$

Q1b Apply the quotient rule, factorise and simplify:

$f'(x)=\frac{-e^x\sin x-e^x\cos x}{e^{2x}}=\frac{-e^x(\sin x+\cos x)}{e^{2x}}=\frac{-(\sin x+\cos x)}{e^x}$

Q2a $\int\frac{3}{2x-3}dx=\frac{3}{2}\log_e(2x-3)+c$

Q2b Definte integral

$=2\int_0^1[f(x)]^2dx-3\int_0^1 f(x)dx=2\times\frac{1}{5}-3\times\frac{1}{3}=-\frac{3}{5}$

Q3 Express in $y=mx+c$ form:

$y=\frac{k}{5}x-\frac{k+4}{5}$ and $y=\frac{-3}{k+8}x-\frac{1}{k+8}$

For infinitely many solutions, equate gradients and equate y-intercepts, .: $k^2+8k+15=0$ AND $k^2+12k+27=0$

Solve simultaneously, only $k=-3$ satisfies both equations.

Q4a Binomial Distribution: $n=4$, $p=\frac{1}{2}$,

$\Pr(X=1)=4\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^3=\frac{1}{4}$, $\Pr(X=3)=\Pr(X=1)=\frac{1}{4}$

$\Pr(X=4)=\Pr(X=0)=\frac{1}{16}$

Q4b Trials are independent of Trial 1: Binomial, $n=3$, $p=\frac{1}{2}$

$\Pr(X=2)=3\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)=\frac{3}{8}$

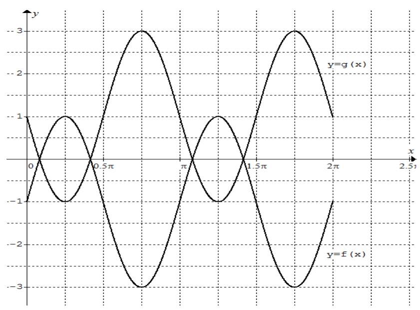
Q4c $n=3$, $p=\frac{2}{3}$, $\Pr(X=2)=3\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)=\frac{4}{9}$

Q5a $10^{3x-13}=10^2$, $3x-13=2$, $x=5$

Q5b For $f(x)$ to be defined, $x^2-2x-3>0$, $(x+1)(x-3)>0$

Maximal domain $R\setminus[-1,3]$

Q6a



Q6b $2\sin 2x-1=0$, $0\le x\le 2\pi$, $\sin 2x=\frac{1}{2}$

$2x=\frac{\pi}{6},\frac{5\pi}{6},\frac{13\pi}{6},\frac{17\pi}{6}$, $x=\frac{\pi}{12},\frac{5\pi}{12},\frac{13\pi}{12},\frac{17\pi}{12}$

Q6ci $g(x)=2\sin 2(x-a)-1+b=-(2\sin 2x-1)$

.: $-1+b=1$, $b=2$

Q6cii $2\sin 2(x-a)=-2\sin 2x$, $\sin(2x-2a)=-\sin 2x$

.: $2a=\pi$, $a=\frac{\pi}{2}$

Q6ciii For $f(x)$, domain $[0,2\pi]$, .: D is $\left[-\frac{\pi}{2},\frac{3\pi}{2}\right]$

Q7ai $20\times 20=400\text{ cm}^2$

Q7aii $a=10$

Q7b $\int_0^{20}g(x)dx=\left[-\frac{x^4}{400}+\frac{x^3}{10}-x^2+10x\right]_0^{20}=200$

The area of the other part of the tile $=400-200=200$.: same area

Q7c For $f(x)$, the end points are $(0,10)$ and $(20,10)$

For $g(x)$, $g(0)=10$ and $g(20)=10$.: the end points are also $(0,10)$ and $(20,10)$. .: when the tiles (A and B types) are joined in the upright position, a continuous curve is formed consisting of $f(x)$ and $g(x)$ in any combinations.

Q8a $A\left(\frac{\pi}{3}\right)=\frac{\pi}{3}\sin\frac{\pi}{3}=\frac{\sqrt{3}\pi}{6}$

Q8b $\int_0^k f(x)dx=A(k)=k\sin k$,

$[F(x)]_0^k=k\sin k$ where $F(x)=\int f(x)dx$.: $f(x)=\frac{d}{dx}F(x)$

$F(k)=k\sin k$ Note: $F(0)=0$

$\frac{d}{dk}\int_0^k f(x)dx=\frac{d}{dk}(k\sin k)$.: $\frac{d}{dk}F(k)=k\cos k+\sin k$

.: $f(k)=k\cos k+\sin k$, $f\left(\frac{\pi}{3}\right)=\frac{\pi}{3}\cos\frac{\pi}{3}+\sin\frac{\pi}{3}=\frac{\pi}{6}+\frac{\sqrt{3}}{2}$

Q8c $Av=\frac{\int_0^k f(x)dx}{k}=\sin k$, $0\le k\le 2$

Av has a maximum value of 1 when $k=\frac{\pi}{2}$.

Please inform mathline@itute.com re conceptual and/or mathematical errors