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2023 Mathematical Methods

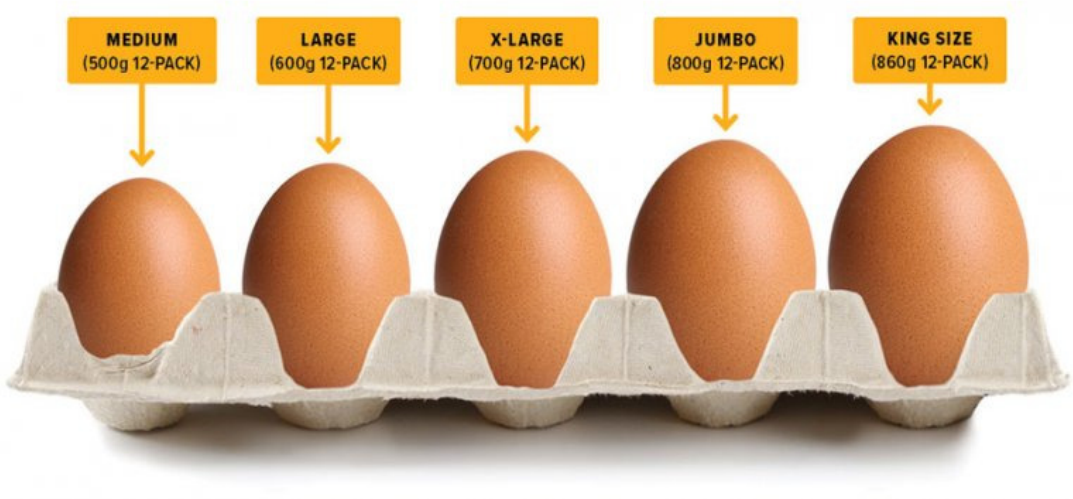
Year 12

Problem Solving Task

Time allowed: 2 hours plus

Problem Solving Task

Theme: Australian egg sizes



Picture from: australianeggs.org.au/farming/egg-types-and-labels

Assumed knowledge:

Discrete and continuous random variables; distributions, binomial distributions, normal distributions and approximation; probability, conditional probability; population proportion, sample proportion, statistical inference; CAS

Information:

Table 1

Category by size (g) for a carton of 12 eggs	Category by description	Weight range (g) of an egg	Mean weight (g) of an egg	Mean weight (g) of edible portion of an egg
500	MEDIUM	< 49.9	43	37
600	LARGE	$50.0 - 58.2$	52	45
700	X-LARGE	$58.3 - 66.6$	60	52
800	JUMBO	$66.7 - 71.6$	68	59
860	KING SIZE	> 71.7	73	64

Part I (60 minutes plus)
Round numerical answers to appropriate number of decimal places unless stated otherwise

Let random variable W be the weight (g) of an egg.

a. Refer to Table 1. Only part of an egg is edible (you do not consume the egg shell).
Calculate the edible proportion of an egg in each category.
Compare and comment.

b. Australian record so far: The largest egg was a 209 g egg from Bungaree Farm near Barwang NSW.
State the value of $\Pr(W = 209)$ and explain your answer.

An egg farmer manages two egg farms, **Farm A** and **Farm B**.

Farm A produces eggs with weight distribution shown in Table 2.

Table 2

Category	MEDIUM	LARGE	X-LARGE	JUMBO	KING SIZE
Mean weight (g) of an egg	43	52	60	68	73
Proportion	0.03	0.20	0.55	0.18	0.04

Farm B produces eggs with weight distribution shown in Table 3.

Table 3

Category	MEDIUM	LARGE	X-LARGE	JUMBO	KING SIZE
Mean weight (g) of an egg	43	52	60	68	73
Proportion	0.02	0.19	0.58	0.17	0.04

c. Calculate the probability of getting 4 X-LARGE eggs in a random sample of 6 eggs taken from Farm A.

d. Calculate the probability of getting 4 X-LARGE and 2 JUMBO eggs in a random sample of 6 eggs taken from Farm A.

e. Calculate the probability of getting more than 4 X-LARGE eggs in a random sample of 6 eggs taken from Farm A.

f. The farmer randomly selects 3 eggs from Farm A and 3 eggs from Farm B.
Calculate the probability of getting 4 X-LARGE eggs.

g. The farmer randomly selects 3 eggs from Farm A and 3 eggs from Farm B.
Calculate the probability of getting 4 X-LARGE and 2 JUMBO eggs.

h. Find the proportion of eggs from Farm A which are at least 58.3 g each in weight.

i. Calculate the probability of getting 4 X-LARGE eggs in a random sample of 6 eggs taken from Farm A which are at least 58.3 g each.

Let random variable W_A be the weight (g) of an egg from Farm A, and W_B the weight (g) of an egg from Farm B.

j. Show that $\overline{W_A} = 59.85$, $\text{sd}(W_A) \approx 6.30$, $\overline{W_B} = 60.02$ and $\text{sd}(W_B) \approx 5.96$

i. Discuss whether random variable weight W (g) of eggs from each farm (Farm A, Farm B) has a normal distribution. Support your discussion with some calculations.

End of Part I

Information from Part I

Table 1

Category by size (g) for a carton of 12 eggs	Category by description	Weight range (g) of an egg	Mean weight (g) of an egg	Mean weight (g) of edible portion of an egg
500	MEDIUM	< 49.9	43	37
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700	X-LARGE	$58.3 - 66.6$	60	52
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An egg farmer manages two egg farms, **Farm A** and **Farm B**.

Farm A produces eggs with weight distribution shown in Table 2.

Table 2

Category	MEDIUM	LARGE	X-LARGE	JUMBO	KING SIZE
Mean weight (g) of an egg	43	52	60	68	73
Proportion	0.03	0.20	0.55	0.18	0.04

Farm B produces eggs with weight distribution shown in Table 3.

Table 3

Category	MEDIUM	LARGE	X-LARGE	JUMBO	KING SIZE
Mean weight (g) of an egg	43	52	60	68	73
Proportion	0.02	0.19	0.58	0.17	0.04

$\overline{W}_{\mathrm{A}} = 59.85$, $\mathrm{sd}(W_{\mathrm{A}}) \approx 6.30$, $\overline{W}_{\mathrm{B}} = 60.02$ and $\mathrm{sd}(W_{\mathrm{B}}) \approx 5.96$ where W_{A} is the weight (g) of an egg from Farm A, and W_{B} the weight (g) of an egg from Farm B.

Part II (60 minutes plus)

Round numerical answers to appropriate number of decimal places unless stated otherwise

Let random variable $\hat{P}$ be the proportion of a sample which are X-LARGE eggs.

a. A random sample of 100 eggs is taken from Farm A.
Find (i) $\mathrm{E}(\hat{P})$, (ii) $\mathrm{sd}(\hat{P})$ and (iii) $\Pr(\hat{P} > 0.57)$ using normal approximation.

b. A random sample of 25 eggs is also taken from Farm B.
Find (i) $\mathrm{E}(\hat{P})$, (ii) $\mathrm{sd}(\hat{P})$ and (iii) $\Pr(\hat{P} > 0.57)$ using normal approximation.

c. Which one of the values of $\Pr\left(\hat{P} > 0.57\right)$ found in part a and part b is more reliable? Explain your answer.

d. Another random sample of n eggs is taken from Farm B.
Find the value of n such that $\Pr\left(\hat{P} > 0.57\right) \approx 0.58$.

e. Another random sample of 100 eggs is taken from Farm A.
The proportion of the sample which are X-LARGE eggs is 0.56.
Assume that the proportion of Farm A eggs which are X-LARGE is unknown.
(i) Estimate the proportion of Farm A eggs which are X-LARGE, and
(ii) determine the approximate 95% confidence interval for the proportion of Farm A eggs which are X-LARGE.

f. A random sample of 100 eggs is to be taken weekly from Farm A for 20 weeks.
The approximate 95% confidence interval for the proportion of Farm A eggs which are X-LARGE is calculated in each case.
Knowing that the proportion of Farm A eggs which are X-LARGE is 0.55, find the expected number of intervals out of 20 containing the proportion 0.55.

g. Explain why the 95% confidence interval calculated in part e is **approximate**.

The ratio number of eggs from Farm A : number of eggs from Farm B is $3:2$.
The egg farmer combines all eggs from both farms.
The weight distribution of the **combined lot** is shown in Table 4.
The prices per dozen of eggs are also shown in the table.

Table 4

Category	MEDIUM	LARGE	X-LARGE	JUMBO	KING SIZE
Mean weight (g) of an egg	43	52	60	68	73
Proportion	0.026	0.196	0.562	0.176	0.040
Price ($) per dozen of eggs	2.90	3.30	3.80	4.20	4.50

h. Show working for the highlighted proportion of the combined lot which are X-LARGE.

i. Calculate the mean weight (g) of a dozen of eggs, i.e. 12, from the combined lot.

j. Calculate the expected returns ($) to the farmer for a dozen of eggs from the combined lot.

End of Part II