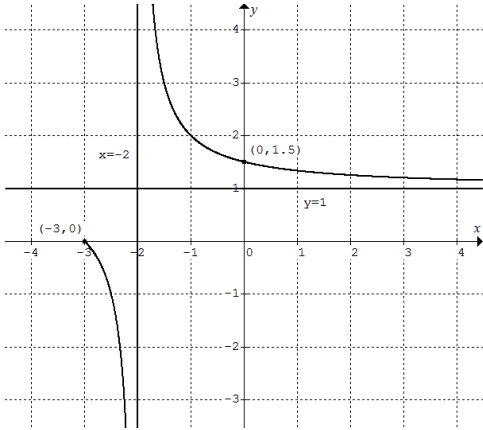


2023 VCAA Mathematical Methods Exam 1 Sample Questions Solutions

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Q1a



Q1b Let $1 + \frac{1}{x+2} = 2$, $x = -1$

$\therefore f(x) \leq 2$ for $x \in [-3, -2) \cup [-1, \infty)$

Q2a $[-9, \infty)$

Q2b $f \circ g(x) = f(g(x)) = (g(x))^2 - 9 = (\sqrt{x})^2 - 9 = x - 9$

Domain of $f \circ g$ is $[0, \infty)$

Q2c $g \circ h(x) = \sqrt{h(x)} \therefore h(x) = x^2 - 9 \geq 0$

$\therefore x \leq -3$ or $x \geq 3$, max domain D is $R \setminus (-3, 3)$

Q3 $2 \sin x = \frac{\sin x}{\cos x}$ and $\cos x \neq 0$

$2 \sin x \cos x - \sin x = 0$, $\sin x(2 \cos x - 1) = 0$

$\therefore \sin x = 0$ or $\cos x = \frac{1}{2}$

$\therefore x = n\pi$ or $\left(2n \pm \frac{1}{3}\right)\pi$ where $n \in J$, set of integers

Q4 $y = -\frac{a+3}{9}x + \frac{b}{3}$ and $y = -\frac{2}{a}x + \frac{5}{a}$

No solutions when $-\frac{a+3}{9} = -\frac{2}{a}$ and $\frac{b}{3} \neq \frac{5}{a}$

$\therefore a = -6$ and $b \neq -\frac{5}{2}$ i.e. $b \in R \setminus \left\{-\frac{5}{2}\right\}$

OR $a = 3$ and $b \neq 5$ i.e. $b \in R \setminus \{5\}$

Q5a Average rate of change = $\frac{f(1) - f(-1)}{1 - (-1)} = 0$

Q5b Average value = $\frac{\int_{-1}^1 (2 - x^2) dx}{1 - (-1)} = \frac{5}{3}$

Q5c Area = $2 \times 0.5 \left(\frac{f(0) + f(0.5)}{2} + \frac{f(0.5) + f(1)}{2} \right) = \frac{13}{4}$

Q6a $f(-1) = -\frac{1}{3} - 2 + 4 > 0$, $f(-2) = -\frac{8}{3} - 4 + 4 < 0$

Q6b $f'(x) = x^2 + 2$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = -1 - \frac{-\frac{1}{3} - 2 + 4}{3} = -\frac{14}{9}$

Q7 $\Pr(T < k) = \int_0^k \frac{2}{5} e^{-\frac{2}{5}t} dt = 0.90 \therefore \left[-e^{-\frac{2}{5}t}\right]_0^k = 0.90$

$-e^{-\frac{2}{5}k} + 1 = 0.90$, $e^{-\frac{2}{5}k} = \frac{1}{10}$, $e^{\frac{2}{5}k} = 10$, $k = \frac{5}{2} \log_e 10$

Please inform mathline@itute.com re conceptual and/or mathematical errors