

2022 VCAA Mathematical Methods Exam 2 Sample Questions Solutions

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SECTION A

Q1 $y = \frac{x-3}{2}$, $z = 2y - 4 = x - 7$

Let $x = k \therefore y = \frac{k-3}{2}$ and $z = k - 7 \quad \forall k \in \mathbb{R}$

Q2 $x_8 \approx x_9 \approx 1.904$

Q3

x	0	1	2	3	4
y	4	$\frac{91}{27}$	$\frac{52}{27}$	1	$\frac{76}{27}$

Q4 Area under $f(x) = \frac{x+1}{20}$ is $\frac{3}{5}$

Area under $f(x) = \frac{36-5x}{64}$ from $x=4$ to $x=a$ is

$\frac{5}{8} - \frac{3}{5} = \frac{1}{40}$ where $4 < a < 7.2$

$f(4) = \frac{1}{4}$, $f(a) = \frac{9}{16} - \frac{5}{64}a$

$\therefore \frac{1}{2}(a-4)\left(\frac{1}{4} + \frac{9}{16} - \frac{5}{64}a\right) = \frac{1}{40}$,

$\frac{1}{2}(a-4)\left(\frac{52-5a}{64}\right) = \frac{1}{40}$, $25a^2 - 360a + 1056 = 0$

$\therefore a = \frac{36 - 4\sqrt{15}}{5}$

Q5 Define trapezium $(\log_e(x), 1, 3, 10)$

$h = \frac{3-1}{10} = 0.2$, $sum = \log_e 3 + \log_e 1 = \log_e 3$,

$x = 1 + 0.2 = 1.2$

While $1 < 10$, $sum = \log_e 3 + 2\log_e 1.2 \approx 1.463$

Q6 Define bisection $(\sin(x), 3, 5, 2)$

$i = 0 < 2$, $mid = (3+5) \div 2 = 4$, $\sin 4 \neq 0$, $\sin 3 \sin 4 < 0$,

$b = 4$
 $i = 1 < 2$, $mid = (3+4) \div 2 = 3.5$, $\sin 4 \neq 0$,

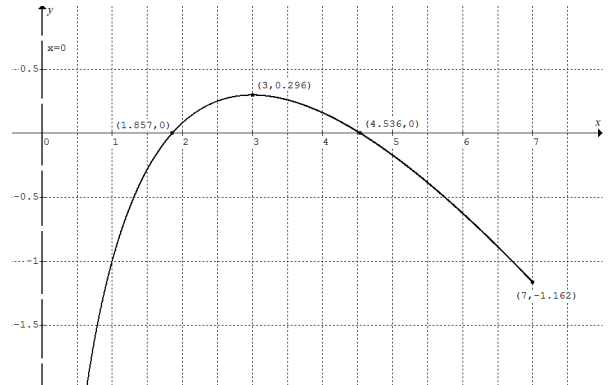
$\sin 3 \sin 3.5 < 0$, $b = 3.5$

Q7 Tolerance = ± 0.001

$\therefore -0.001 < next_x - prev_x < 0.001$

SECTION B

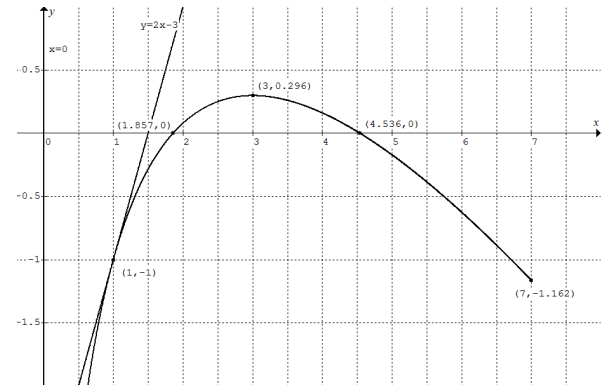
Q1a



Q1bi $g(1) = -1$, $g'(x) = \frac{3}{x} - 1$, $g'(1) = 2$

\therefore tangent is $y = 2x - 3$

Q1bii



Q1c $x_1 = 1 - \frac{-1}{2} = \frac{3}{2}$

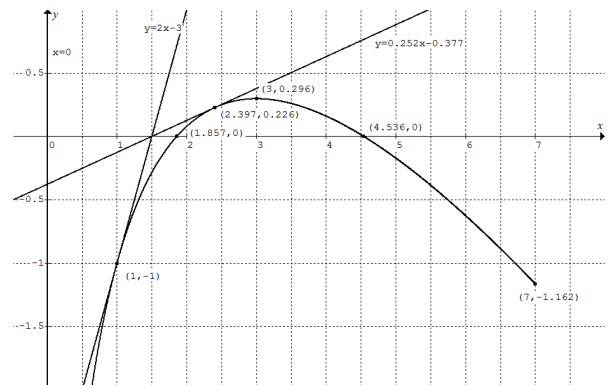
Q1d $x_2 \approx 1.783604676$, $x_3 \approx 1.85354065$

x -intercept, $x \approx 1.85718$,
horizontal distance $\approx 1.85718 - 1.85354 \approx 0.0036$

Q1ei $k - \frac{3\log_e k - k}{\frac{3}{k} - 1} = \frac{3}{2}$ where $k > 1 \therefore k \approx 2.397$

Q1eii $g(2.397) \approx 0.22565$, $g'(2.397) \approx 0.25156$

\therefore tangent is $y = 0.25156x - 0.3773$





Q2a

$$f'(x) = \begin{cases} 0 & 0 < x < 5 \\ \frac{1}{800}(3x^2 - 150x + 675) & 5 \leq x < 55 \end{cases}$$

Q2bi $f''(x) = 0$, $x = 25$ and $y = 20$,
point of inflection is $(25, 20)$

Q2bii $[25, 55]$

Q2biii $[5, 25]$

Q2c Estimate:

$$\frac{1}{2} \times 10 \times (40 + 33.75) + \frac{1}{2} \times 10 \times (33.75 + 20) = 637.5$$

$$\text{Exact: } \int_5^{25} \frac{1}{800}(x^3 - 75x^2 + 675x + 30375) dx = 650$$

Ratio *estimate:exact* is $637.5 : 650 = 98.1\%$

Q2d The gradient of the curve is increasing in the interval $[25, q]$, i.e., the curve is concave up \therefore the top edges of the trapeziums are higher than the curve $y = f(x)$.

$$\text{Q2ei } g(55) = f(55) \therefore 55b + c = 197.8125$$

$$g'(55) = f'(55) \therefore -6.825 + b = 1.875$$

$$\therefore b = 8.75 \text{ and } c = -283.4375$$

Q2eii $a > 55$ and

$$P(a) = g(a) = -\frac{1}{16}a^2 + 8.75a - 283.4375 = 0$$

$$\therefore a \approx 89.10$$

and horizontal distance $\approx 89.10 - 55 = 34.10$ cm

Please inform admin@itute.com re conceptual and/or mathematical errors