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Mathematical Methods

2024

Trial Examination 1 (1 hour)

Instructions

Answer **all** questions.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1 (6 marks)

Consider $f(x) = 2x^2 + 1$ where $x \in [-1, 2]$.

The graph of $f(x)$ is dilated in both x and y directions by the same factor of 3, resulting in the graph of $g(x)$.

a. State the domain and range of $g(x)$. 1 mark

b. Let $g(x) = ax^2 + b$. Find the values of a and b . 2 marks

c. Show algebraically that $g(x)$ is transformed back to $f(x)$ after it is dilated in both x and y directions by the same factor of $\frac{1}{3}$.

3 marks

Question 2 (6 marks)

a. Use two of the transformations *dilation*, *reflection* and *translation* to show the symmetry of $f(x) = \sin(2x)$ where $x \in \mathbb{R}$. Write the equation to show the symmetry.

2 marks

b. Solve $\sin\left(2x - \frac{\pi}{4}\right) + \sin\left(2x + \frac{3\pi}{4}\right) = 0$ for x . 2 marks

c. Solve $\sin\left(\frac{\pi}{2} - 2x\right) + \sin\left(\frac{\pi}{2} - 3x\right) = 2$ for x . 2 marks

Question 3 (6 marks)

Let $f(x) = \log_e(kx^n)$ where $k \in R^+$ and $n \in Q$, and $f^{-1}(x) = e^{a(x-b)}$.

a. Find a and b in terms of k and n .

2 marks

b. Show that the graphs of $f(x)$ and $f^{-1}(x)$ intersect at exactly one point when $k = \left(\frac{e}{n}\right)^n$.

2 marks

c. In terms of k and n find the coordinates of the intersection point in part b.

2 marks

Question 4 (5 marks)

Line $y = mx + c$ passes through the point $(-1, 2)$ and the point of inflection of $y = f(x)$ where

$$f(x) = \frac{3\sqrt{3}}{2}(x^3 - x) + 1.$$

a. Find the coordinates of the stationary points of $y = f(x)$. State the type of each stationary point and justify your answers.

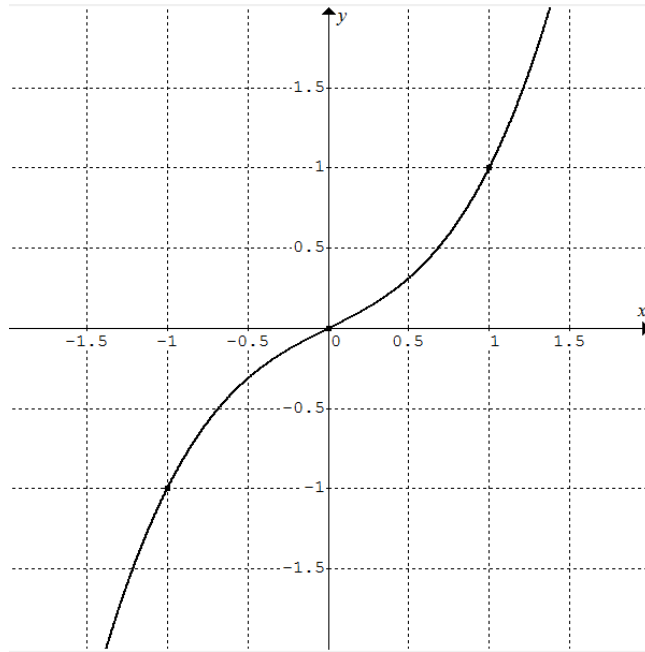
3 marks

b. Find the coordinates of the intersections of $y = f(x)$ and $y = mx + c$.

2 marks

Question 5 (7 marks)

The graph of $y = \frac{1}{2}x(x^2 + 1)$ is shown below.



a. Given that $\tan\left(\frac{\pi x}{4}\right) > \frac{1}{2}x(x^2 + 1)$ when $0 < x < 1$, sketch the graph of $y = \tan\left(\frac{\pi x}{4}\right)$ for $-1 \leq x \leq 1$ on the grid shown above.

2 marks

b. Given $F(x) = -\frac{4}{\pi} \log_e \left(\cos\left(\frac{\pi x}{4}\right) \right)$ for $-1 \leq x \leq 1$, show that $F'(x) = \tan\left(\frac{\pi x}{4}\right)$.

2 marks

c. Find the total area of the regions enclosed by the graphs of $y = \frac{1}{2}x(x^2 + 1)$ and $y = \tan\left(\frac{\pi x}{4}\right)$.

3 marks

Question 6 (5 marks)

Three cubic dice are rolled. The faces of each die are numbered from 1 to 6.

a. How many elements are there in the sample space of rolling the three cubic dice?

1 mark

b. Consider event A : Exactly two dice show an odd number less than 5 on the uppermost face. Find the value of $\Pr(A)$.

1 mark

Let random variable X be the number of odd numbers less than 5 appearing on the uppermost face.

c. Find the value of $\Pr(X \leq 1)$.

1 mark

d. Find the value of $\Pr(X = 1 | X \leq 2)$.

1 mark

e. Find the mean value of X .

1 mark

Question 7 (5 marks)

A size 75 and a size 300 samples were taken from a population of people aged 25 to 35.

Number of people requiring glasses for sight correction is 15 in the smaller sample and 75 in the larger sample.

Useful information:

For normal distributions, approximately 68% of data lies within 1 standard deviation from the mean and 95% of data lies within 2 standard deviations from the mean.

a. Calculate the proportion of people requiring glasses in each sample. 1 mark

b. Based on the two samples taken calculate the best estimate of the 95% confidence interval for the population proportion requiring glasses, correct to 1 decimal place. 2 marks

c. If 75 samples of size 300 were taken, estimate the number of samples having sample proportion greater than 0.3 to the nearest whole number. 1 mark

d. If 300 samples of size 75 were taken, calculate the best estimate of the number of samples having sample proportion greater than 0.3 to the nearest whole number. 1 mark

End of Examination 1