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2024 Mathematical Methods

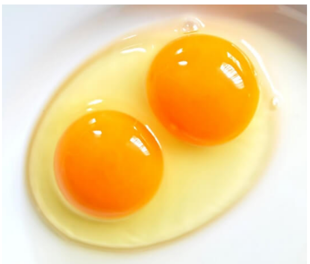
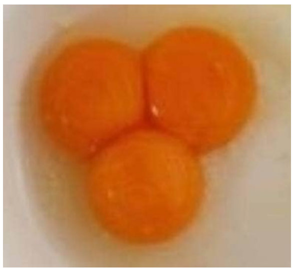
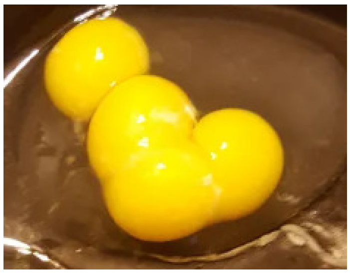
Year 12

Problem Solving Task

Time allowed: 2 hours plus

Problem Solving Task

Theme: Multi yolk eggs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Assumed knowledge:

Discrete and continuous random variables; distributions, binomial distributions, normal distributions and approximation; probability, conditional probability; population proportion, sample proportion, statistical inference; CAS

Information:

Estimated chance of a double yolk egg to happen is one in every 1000 eggs, and a triple yolk to happen one in every 25 million eggs.
There are no official odds for a four yolk egg.

Part I (60 minutes plus)
Round numerical answers to appropriate number of decimal places unless stated otherwise

a. The probability of success in a single Bernoulli trial is $\frac{1}{5}$.

Random variable X is defined as the ***number of successes*** in five Bernoulli trials.

i. How many possible outcomes are there in the sample space of the five Bernoulli trials? Use S for success and F for failure, write down two of the possible outcomes.

ii. Show that the probability of one success in five Bernoulli trials is 0.4096.

iii. Set up a table to display the probability distribution of random variable X in five Bernoulli trials.

iv. Calculate the probability of one success in five Bernoulli trials, given that the success does not occur in the first two trials.

b. The probability of success in a single Bernoulli trial is $\frac{1}{5}$.

Trials are performed **until there is a success**. Random variable X is defined as the number of trials performed.

The sample space is composed of simple events. Each event consists of a number of F's (failures) followed by a single S (success).

For $X = 1,\ 2$ and 3, the corresponding events are S, FS and FFS respectively.

i. Write down the next two events for $X = 4$ and 5.

ii. Complete the following table. Show that the sum of the five probabilities is 0.67232.

x	1	2	3	4	5
$\Pr(X = x)$	0.2	0.16			0.08192

iii. Give two reasons why the table above is not a probability distribution of X.

The **sum** of the first five probabilities in the table above is 0.67232.

It represents the probability of a success in five trials; either the success occurs in the first trial, the second trial, the third trial, the fourth trial or the fifth trial.

iv. Explain why this probability is greater than the probability of one success in five Bernoulli trials discussed in part a ii.

c. Let the chance of picking a double yolk egg be one in every 1000 eggs.
Eggs are selected randomly one at a time.

i. Calculate the probability of getting no double yolk egg in 1000 randomly selected eggs.

ii. Calculate the probability of getting exactly one double yolk egg in 1000 randomly selected eggs.

iii. Calculate the probability of getting at least one double yolk egg in 1000 randomly selected eggs.

iv. Calculate the probability of getting exactly two double yolk egg in 1000 randomly selected eggs.

v. Given that exactly a double yolk egg is picked in the first five random selections, calculate the probability of getting exactly one double yolk egg in the next 995 randomly selected eggs.

d. Let the chance of picking a double yolk egg be one in every 1000 eggs.
Eggs are selected one at a time **until a double yolk egg is picked**.
Let D be double yolk egg in selecting an egg, and D' not double yolk egg. $\Pr(D) = 0.001$

i. Find $\Pr(D) + \Pr(D'D) + \Pr(D'D'D)$.

ii. Find a general expression in terms of n for $\Pr(D'D'D'\cdots D'D)$ where D occurs in the n^{th} selection.

iii. Given the formula $1+x+x^2+\cdots+x^{n-1}=\dfrac{1-x^n}{1-x}$, express $\Pr(D)+\Pr(D'D)+\Pr(D'D'D)+\cdots+\Pr(D'D'D'\cdots D'D)$ in simplified form in terms of n.

iv. Find n such that the probability of picking a double yolk egg in the first selection, the second selection, the third selection, … or the n^{th} selection is closest to 0.5.

v. Show that it is certain to pick a double yolk egg eventually.

End of Part I

Part II (60 minutes plus)
Round numerical answers to appropriate number of decimal places unless stated otherwise

Information

Estimated chance of a double yolk egg to happen is one in every 1000 eggs, and a triple yolk to happen one in every 25 million eggs.

Egg farmers sort out double yolk eggs by a process called ‘candling’ which involves holding an egg up to the light and seeing through the shell.

This is a routine procedure done by egg farmers. It allows them to market cartons of double yolk eggs.

A supermarket chain promotes its home brand eggs using the gimmick that a customer has a greater chance of finding double yolk eggs than those in other supermarket chains.
An egg farm agrees to provide the supermarket chain weekly delivery of eggs, with the ratio ***single yolk eggs*** : ***double yolk eggs*** being $5:1$, randomly packed in cartons of 12 eggs.

a. Assume that the egg farm honouring the agreement, determine the proportion of double yolk eggs in a delivery.

b. Fill in the blanks in the following probability distribution of proportion $\hat{P}$ of double yolk eggs in a carton of the home brand eggs, using the binomial distribution of the number X of double yolk eggs in a carton.

x	0	1	2	3	4	5	6	7	8	9	10	11	12
$\hat{p}$	0			0.25			0.5			0.75			1
$\Pr(\hat{P}=\hat{p})$	$\left(\frac{5}{6}\right)^{12}$			$220\left(\frac{1}{6}\right)^{3}\left(\frac{5}{6}\right)^{9}$			$924\left(\frac{1}{6}\right)^{6}\left(\frac{5}{6}\right)^{6}$			$220\left(\frac{1}{6}\right)^{9}\left(\frac{5}{6}\right)^{3}$			$\left(\frac{1}{6}\right)^{12}$

c. By considering the different possibilities in getting 4 double yolk eggs in 2 cartons of eggs, determine the probability of getting 4 double yolk eggs in 2 cartons of eggs.

d. Find the probability of getting 4 double yolk eggs in 24 eggs. Is this answer the same as that in part c. Discuss/explain

e. Consider each carton as a random sample of 12 eggs.

(i) **Use the completed table in part b** to evaluate $\Pr(0.25 < \hat{P} < 0.5)$.

(ii) **Use the completed table in part b** to evaluate $\mathrm{E}(\hat{P})$ and $\mathrm{sd}(\hat{P})$, i.e. the expectation and standard deviation of the proportion of double yolk eggs in a carton of the home brand eggs. Verify that $\mathrm{E}(\hat{P}) = p$ and $\mathrm{sd}(\hat{P}) = \sqrt{\frac{p(1-p)}{n}}$.

(iii) Use normal approximation to evaluate $\Pr(0.25 < \hat{P} < 0.5)$.

(iv) Comment and explain the difference in the answers to e(i) and e(iii) in terms of the shapes of the distributions.

f. Consider any ten cartons as a random sample of 120 eggs. Evaluate $\Pr\left(0.25 < \hat{P} < 0.5\right)$ using (i) binomial distribution and (ii) normal approximation.
Explain why the probability values are less than the corresponding values in part e.
Explain why the difference of these two probability values is less than the difference of the two values in part e.

Some customers of the supermarket chain complain that they do not get that many double yolk eggs in a carton as they are made to believe. The management suspects that the egg farm does not meet the requirement in the agreement to provide weekly delivery of eggs, with the ratio ***single yolk eggs*** : ***double yolk eggs*** being $5:1$, randomly packed in cartons of 12 eggs.

g. The management randomly selects ten cartons of eggs (120 eggs) from a delivery and find that there are 16 double yolk eggs.
Determine the 95% confidence interval for the proportion of double yolk eggs in the delivery. (Assume that the 120 eggs from the ten cartons constitute a random sample.)
Comment and explain whether the agreement is honoured by the egg farmer.

h. Comment on the validity of the assumption in part g.

End of Part II