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Specialist Mathematics

2024

Trial Examination 1
(1 hour)

Instructions

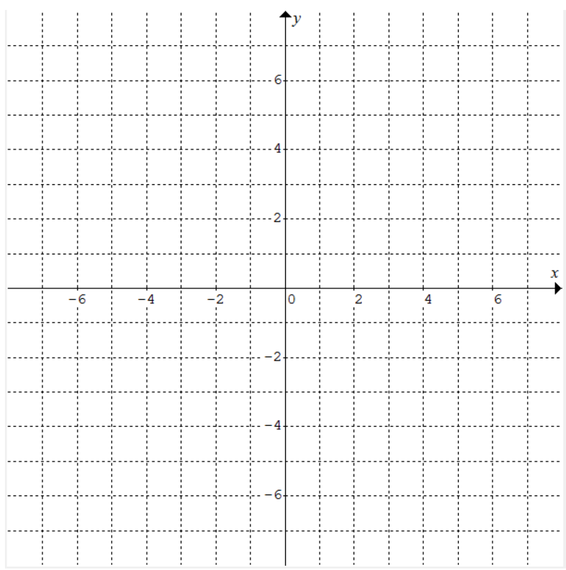
Answer **all** questions. Do **not** use calculators.
Unless otherwise specified, an **exact** answer is required to a question.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
In questions where more than one mark is available, show appropriate working or explanation.
Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$

Question 1 (5 marks)

Let $f(x) = \dfrac{x^3 - 2x^2 + x}{x^3 - x^2 - x + 1} = a + \dfrac{b}{x+c}$.

a. Evaluate a, b and c. 2 marks

b. Sketch the graph of $y = f(x)$. Show and label all key features. 3 marks



Question 2 (4 marks)

Solve the following equations for x over C.

a. $4x^4 + 5x^2 + 1 = 0$. 2 marks

b. $4x^4 + 3x^2 + 1 = 0$. 2 marks

Question 3 (7 marks)

The position vectors of particles A and B are $\underset{\sim}{r}_A = (2-t)\underset{\sim}{j} + (t-1)\underset{\sim}{k}$ and $\underset{\sim}{r}_B = t\underset{\sim}{i} + (2-t)\underset{\sim}{j}$ respectively for $t \geq 0$.

a. Show that the path of A lies on the plane $x + y + z = 1$, and the path of B lies on the plane $x + y + z = 2$.

2 marks

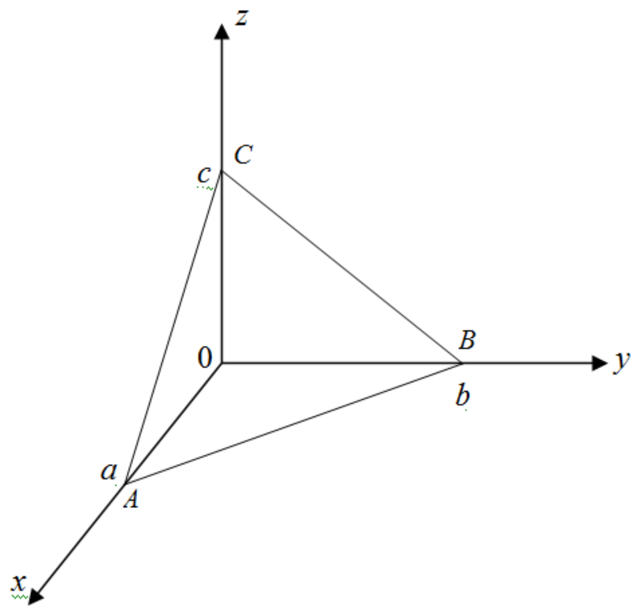
b. Find the distance between the two particles at $t \geq 0$. Hence find the closest approach of the two particles and the time when it occurs.

3 marks

c. Calculate the shortest distance between the path of particle A and the path of particle B.. 2 marks

Question 4 (4 marks)

$(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$ are vertices of ΔABC on the orthogonal axes *x*, *y* and *z*.



a. In terms of *a*, *b* and *c*, determine the area of ΔABC. 2 marks

b. In terms of *a*, *b* and *c*, determine the acute angle between the planes defined by ΔOAB and ΔABC, where O is the origin of the orthogonal axes *x*, *y* and *z*.

2 marks

Question 5 (5 marks)

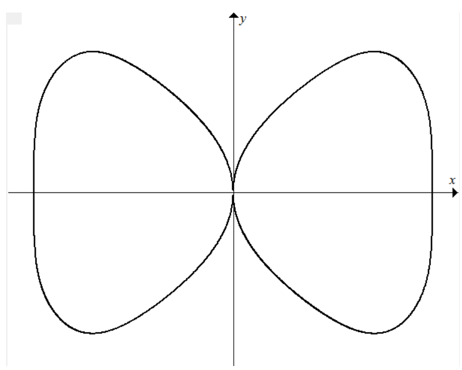
a. i. Expand $(x-1)(x^{n-1}+x^{n-2}+x^{n-3}+\cdots+x^2+x^1+x^0)$ where n is an integer greater than 1. 1 mark

a. ii. Hence show that $2^n = 2^{n-1}+2^{n-2}+2^{n-3}+\cdots+2^2+2^1+2^0$. 2 marks

b. Given that n is not the square of a positive integer, prove by contradiction that $\sqrt{n} \neq \frac{h}{k}$ where h and k are positive integers. 2 marks

Question 6 (3 marks)

The graph of $\sin(x^2)+\cos(y^2)=1$ for $-2<x<2$ is shown below.

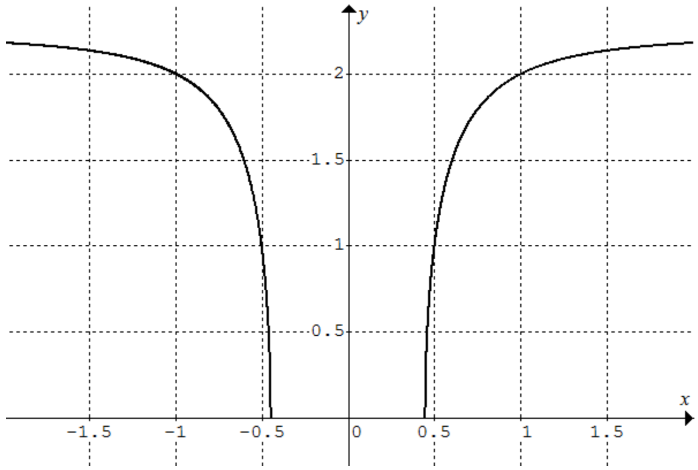


a. Determine the exact positive x-intercept. 1 mark

b. Determine the exact coordinates of the stationary point in the second quadrant. 2 marks

Question 7 (5 marks)

The graph of the *inverse* of $y = \pm\dfrac{1}{\sqrt{5-x^2}}$ is shown below.



a. Find the exact value of the area of the region bounded by the two parts of the graph from $y = 0$ to $y = 2$.

2 marks

b. Find the exact volume of the solid formed when the region described in part a is rotated about the y-axis.

3 marks

Question 8 (3 marks)

Let p be the proportion of women over 18 years old in a population. The mean weight of this group of women in the population is 79 kg, and the mean weight of the remaining over 18 people in the population is 84 kg. A sample of 625 over 18 people was taken and the mean weight of a person in the sample was 81 kg.

a. Estimate the value of p. 1 mark

b. Given the standard deviation of the weight of over 18 in the population is 10 kg, estimate the number of samples having a mean weight between 81 kg and 82 kg, correct to 1 decimal place.

2 marks

Question 9 (4 marks)

A particle moves in a straight line. It has an acceleration $a = \log_e x$ where $x > 0$ is the particle position.

a. Write a suitable differential equation which gives speed v in terms of x when it is solved. 1 mark

b. Given $v = 2$ when $x = 1$, solve the differential equation in part a for speed v in terms of position x.

3 marks

End of Exam 1

