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2024 Specialist Mathematics

Year 12 Application Task

(Time allowed: 4 hours plus)

Theme: Investigate simple rational functions of a real variable and key features of their graphs

Assumed knowledge: Algebra, functions and graphs, parameters, calculus and CAS

Requirements: For each function/graph, identify maximal domain/range, and key features such as axis intercepts, asymptotes, stationary points, points of inflection and symmetry.
Specify/label key features with coordinates/equations.
Draw neat graphs and scale each axis appropriately to show the key features clearly.

Part I (80 min plus)

Consider the function with rule $f(x)=\dfrac{1}{x^3-1}$.

a. By hand find the first and second derivatives of $f(x)$.

b. Identify the key features of $f(x)$.

c. Hence draw the graph of $f(x)$ and label its key features.

Consider the function with rule $h(x)=\dfrac{1}{x^3+1}$.

d. Analyse $h(x)$ by repeating parts a, b and c.

Study the graphs of $f(x)$ and $h(x)$.

e. Statement: $f(x)$ and $h(x)$ show symmetry under certain transformations.
Discuss the meaning of the statement. Demonstrate the statement algebraically.

Consider $f_n(x)=\frac{1}{x^3-n}$ where $n\in R$.

f. In terms of n specify the key features of $f_n(x)$.

A family of functions can be generated by varying the value of parameter n.

g. Systematically choose seven representative and suitable values of parameter n to illustrate the family of curves (show graphs). Label each curve with its n value. Specify/label the key features. Discuss the changes in the key features when the n value changes.

End of Part I

Part II (80 min plus)

Requirements: For each function/graph, identify maximal domain and range, and key features such as axis intercepts, asymptotes, stationary points and points of inflection. Specify key features with coordinates/equations. Draw neat graphs and scale each axis appropriately to show the key features clearly.

Consider $g_n(x) = \dfrac{x}{x^3 - n}$, where $n \in R$.

a. By hand find the first and second derivatives of $g_n(x)$. Identify the key features of $g_n(x)$.
Hence draw the graph of $g_n(x)$ and specify/label its key features, CAS allowed.

Consider $g_m = \dfrac{x^m}{x^3 - 1}$, where m is a **positive integer**. A family of curves can be generated by varying the value of parameter m.

b. Systematically choose six representative and suitable values of parameter m (even and odd) to illustrate the family of curves (show neat graphs, CAS allowed). Label each curve with its m value. Specify/label the key features. Discuss the changes in the key features when the m value changes.
Comment on the effect of **even/odd** m on the appearance of the curve.

c. Compare the graphs of $g_m = \dfrac{x^m}{x^3 - 1}$ when m is a **negative integer** with the graphs of $g_m = \dfrac{x^m}{x^3 - 1}$ when m is a **positive integer** as in part c. Illustrate with $m = \pm 2$.

Consider $g_k = \dfrac{x-k}{x^3-1}$, where $k \in R$. A family of curves can be generated by varying the value of parameter k.

d. Systematically choose seven representative and suitable values of parameter k to illustrate the family of curves (show graphs, CAS allowed). Label each curve with its k value. Specify/label the key features. Discuss the changes in the key features when the k value changes.

e. The graph of $g_k = \dfrac{x-k}{x^3-1}$ for a particular value of $k \in R$ separates the family of curves into two groups with distinct appearances. What is that particular value of k?

Express $g_k(x)$ in simplest form for that particular value of k

End of Part II

Part III (80 min plus)

Requirements: For each function/graph, identify maximal domain and range, and key features such as axis intercepts, asymptotes, stationary points and points of inflection. Specify key features with coordinates/equations. Draw neat graphs and scale each axis appropriately to show the key features clearly.
Use similar procedures in Part I and Part II to investigate graphs of the following functions and their families.

Consider $h_n(x)=\dfrac{x^2+x+n}{x^3-1}$ where $n\in R$.

a. Investigate/analyse the graphs of functions generated by $h_n(x)=\dfrac{x^2+x+n}{x^3-1}$, $n\in R$.

Identify two particular values of n which distinguish the graphs of $h_n(x)=\dfrac{x^2+x+n}{x^3-1}$ from the rest of the family.

Discuss and explain the appearances of the two graphs.

Consider $h_{n,k}(x)=\dfrac{x^3+n}{x^2+k}$ where $k,n\in R$.

b. Investigate/analyse the graphs of functions generated by $h_{n,k}(x)=\dfrac{x^3+n}{x^2+k}$, $k,n\in R$.

Hint: Select a suitable value for k whilst the value of n is varied. Select a suitable value for n whilst the value of k is varied.
Identify any particular k or n values of interest.

End of Part III
End of Task