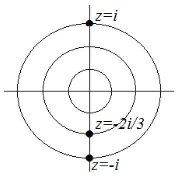
2024 VCAA Specialist Mathematics Exam 1 Solutions

Q1a $f(z)=(3z^3+2iz^2)+(3z+2i)=z^2(3z+2i)+(3z+2i)$
$=(z^2+1)(3z+2i)$

Q1b $f(z)=0$, $3z+2i=0$ or $z^2+1=0$ $\therefore$ $z=-\dfrac{2i}{3},-i$ or i

Q1c



Q2 $x=2n+1,\ n\in Q$

$\therefore$ $2x^2-3x-7=2x^2-3(2n+1)-7=2x^2-6n-10=2(x^2-3n-5)$

$\therefore$ $2x^2-3x-7$ is even.

Q3a $A(x+1)^2+B(x+1)+C=(x-1)^2$ where $x\neq -1$

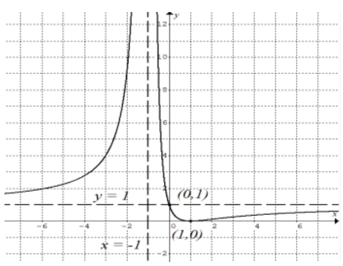
Let $x=1$, $4A+2B+C=0$, let $x=0$, $A+B+C=1$
let $x=-2$, $A-B+C=9$ $\therefore$ $A=1$, $B=-4$ and $C=4$

Q3b $f'(x)=\dfrac{2(x+1)^2(x-1)-2(x-1)^2(x+1)}{(x+1)^4}=0$

$\therefore$ $(x+1)(x-1)-(x-1)^2=0$ $\therefore$ $x-1=0$ $\therefore$ $x=1$ and $y=f(1)=0$

Turning point is $(1,0)$.

Q3c



Q4a $\theta=\cos^{-1}\left(\dfrac{\tilde{a}\cdot\tilde{b}}{|\tilde{a}||\tilde{b}|}\right)=\cos^{-1}\left(\dfrac{-1}{\sqrt{2}}\right)=\dfrac{3\pi}{4}$

Q4b $\tilde{a}\times\tilde{c}=-3\tilde{i}+3n\tilde{j}-3n\tilde{k}$, $|\tilde{a}\times\tilde{c}|=3\sqrt{1+2n^2}$, $\tilde{a}\cdot\tilde{c}=9$

Let $3\sqrt{1+2n^2}=9$ $\therefore$ $1+2n^2=9$, $n=\pm 2$

Q5 $\displaystyle\int_1^{\frac{k}{2}}\pi\left(k-\frac{1}{x^2}\right)dx=\pi\left[kx+\frac{1}{x}\right]_1^{\frac{k}{2}}=\pi\left(\frac{k^2}{2}+\frac{2}{k}-k-1\right)=\frac{7\pi}{2}$

$\therefore$ $k^3-2k^2-9k+4=0$

Q6a Mean $=1.0+1.5+2.0=4.5$, Var $=0.3^2+0.4^2+0.5^2=0.5$

Q6b Var $=(0.3\times 10)^2+(0.4\times 20)^2+(0.5\times 15)^2=129.25$

Q6c $\Pr(W_2<W_1)=\Pr(W_2-W_1<0)=\Pr(X<0)$;

$\bar{X}=1.5-1.0=0.5$, $\sigma_X=\sqrt{0.4^2+0.3^2}=0.5$ $\therefore$ $z=\dfrac{0-0.5}{0.5}=-1$

$\Pr(X<0)=\Pr(Z<-1)=\dfrac{1-0.68}{2}=0.16$

Q7 $2y\dfrac{dy}{dx}=-\dfrac{x}{\sqrt{x^2+1}}$, $\displaystyle\int 2y\frac{dy}{dx}dx=-\int\frac{x}{\sqrt{x^2+1}}dx$

Let $u=x^2+1$, $\displaystyle\int 2y\frac{dy}{dx}dx=-\frac{1}{2}\int\frac{1}{\sqrt{u}}du$

$\therefore$ $y^2=-\sqrt{u}+c=-\sqrt{x^2+1}+c$ and $y(0)=-2$ $\therefore$ $c=5$

$\therefore$ $y^2=5-\sqrt{x^2+1}$ $\therefore$ $y=-\sqrt{5-\sqrt{x^2+1}}$

Q8a $\dfrac{d}{dx}(x^2y^2+xy)=0$, $2xy^2+x^2 2y\dfrac{dy}{dx}+y+x\dfrac{dy}{dx}=0$

$y(2xy+1)+x(2xy+1)\dfrac{dy}{dx}=0$, $\dfrac{dy}{dx}=-\dfrac{y(2xy+1)}{x(2xy+1)}$ $\therefore$ $\dfrac{dy}{dx}=-\dfrac{y}{x}$

Q8b $\dfrac{dy}{dx}=-\dfrac{y}{x}=-1$ $\therefore$ $y=x$, $x^4+x^2=2$ $\therefore$ $(x^2-1)(x^2+2)=0$

$\therefore$ $x^2-1=0$ $\therefore$ $x=-1,\ y=-1;\ x=1,\ y=1$

The points are $(-1,-1)$ and $(1,1)$

Q9a $v=40+10\%$ of $40=44$, $v^2=1936$ which occurs when $x=0$.

When $-15\le x<0$, $v^2>1936$, i.e. $v>44$ device will be activated.

Q9b $a=\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)=\dfrac{d}{dx}\left(800+\dfrac{336}{\pi}\cos^{-1}\left(\dfrac{x}{20}\right)\right)=-\dfrac{336}{\pi\sqrt{20^2-x^2}}$

When $x=12$, $a=-\dfrac{336}{\pi\sqrt{20^2-12^2}}=-\dfrac{21}{\pi}$

Q10 Let $\tilde{a}=\tilde{i}+2\tilde{j}+\tilde{k}$ and $\tilde{b}=-\tilde{i}+3\tilde{j}+2\tilde{k}$

Vectors perpendicular to both vectors are $\tilde{p}=\pm\tilde{a}\times\tilde{b}=\pm(\tilde{i}-3\tilde{j}+5\tilde{k})$

and $\hat{p}=\pm\dfrac{1}{\sqrt{35}}(\tilde{i}-3\tilde{j}+5\tilde{k})$.

$\tilde{r}_1(0)=\tilde{i}+m\tilde{k}$, $\tilde{r}_2(0)=2\tilde{i}-\tilde{k}$.

Let $\tilde{q}=\tilde{r}_1(0)-\tilde{r}_2(0)=-\tilde{i}+(m+1)\tilde{k}$

Scalar resolute of $\tilde{q}$ in the direction of $\hat{p}$ gives the shortest distance.

$\therefore$ $\tilde{q}\cdot\hat{p}=\pm\dfrac{1}{\sqrt{35}}(-1+5(m+1))=\dfrac{14}{\sqrt{35}}$, $\therefore$ $m=2$ or $m=-\dfrac{18}{5}$

Please inform mathline@itute.com re conceptual and/or mathematical errors.