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2026 Mathematical Methods

Year 12

Modeling Task

Time allowed: 2 hours plus

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Modeling Task

Theme: Highways across four regions

Introduction:

Network of highways (assumed to be horizontal) can be modeled using mathematical functions. They can intersect or join smoothly.

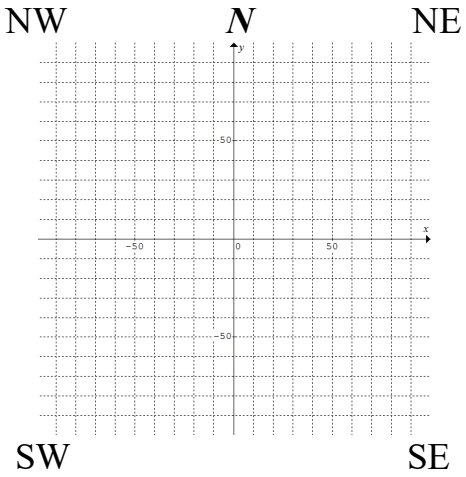
Assumed knowledge:

Functions and relations, quadratic and polynomial functions; graph plotting and sketching; algebraic skills; equations and simultaneous equations; tangent and normal; distance and direction; differential and integral calculus: CAS

Part I (60 minutes plus)

Measurements are in kilometers (km).

Let x and y axes divide the land into four regions, NW region, NE region, SW region and SE region. ***N*** represents direction north.



A highway crossing the SW and SE can be modeled by a quadratic function with equation of the form $y = ax^2 + bx + c$. It passes through four towns with coordinates $(30, -9)$, $(-10, -1)$, $(60, -36)$ and $(40, -16)$.

(1) Show that $a = -\frac{1}{100}$, $b = 0$ $c = 0$.

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Another highway crossing regions can be modeled by quartic polynomial $y = \frac{1}{10000}(x-10)(x-20)(x-30)(x-60)$. It passes through five border towns.

(2) Write down the coordinates of the five border towns.

(3) Find the coordinates of the southernmost point on this highway, correct to 1 decimal place.

(4) Find the coordinates of the points where the two highways meet, correct to 2 decimal places.

(5) Sketch the two highways on the same axes showing the points discussed earlier with labeled coordinates.

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(6) Find the exact straight line distance between the border towns which are the most apart.

(7) Find the coordinates of the point on the 'quadratic' highway where a motorist travels in the ***SE*** direction, correct to 2 decimal places.

(8) Find the area (km^2) of the land enclosed by the two highways, correct to the nearest whole number.

End of Part I

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Part II (60 minutes plus)

(9) You have found the coordinates (corrected to 2 decimal places) of the points where the two highways meet.
Follow the following steps in solving the equations for the two highways simultaneously to find the **exact** coordinates.

The equations are $y = -\frac{1}{100}x^2$ and $y = \frac{1}{10000}(x-10)(x-20)(x-30)(x-60)$.

Step 1: Show $(x-10)(x-60)(x-20)(x-30)+100x^2 = 0$.

Step 2: Expand $(x-10)(x-60)$ and $(x-20)(x-30)$ to form quadratic factors $Q_1(x)$ and $Q_2(x)$.

Step 3: Divide the equation in Step 1 by x^2 to obtain $\frac{Q_1(x)}{x}.\frac{Q_2(x)}{x} - 100 = 0$ where $\frac{Q_1(x)}{x} = x - 70 + \frac{600}{x}$. Express $\frac{Q_2(x)}{x}$ in the same form as $\frac{Q_1(x)}{x}$.

Step 4: Let $u = x + \frac{600}{x}$. Show that $u^2 - 120u + 3600 = 0$.

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Step 5: Solve for u.

Step 6: Hence show that $x = 10\left(3 \pm \sqrt{3}\right)$.

Step 7: Find the exact coordinates of the meeting points. Evaluate each to 2 decimal places for comparison with your answers in Part I (4).

At the two highways meeting point **nearest** to the borders, there is an overhead pedestrian crossing bridge. Consider the crossing bridge as a straight line normal to the highways at the meeting point.

(10) Determine the exact equation of the line representing the crossing bridge.

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(11) The direction of this crossing bridge is $N\theta°E$ (equivalent to $S\theta°W$).
Find θ, correct to 1 decimal place.

A town has coordinates $(30, -30)$.

(12) Determine the shortest distance of this town from a highway.

End of Part II
End of Modeling Task

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