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2026 Specialist Mathematics

Year 12 Modelling Task

(Time allowed: 2.0 hours plus)

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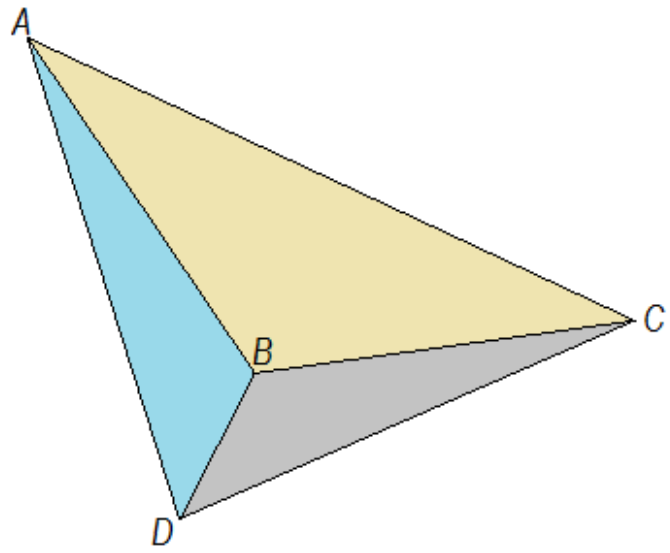
Modelling Task

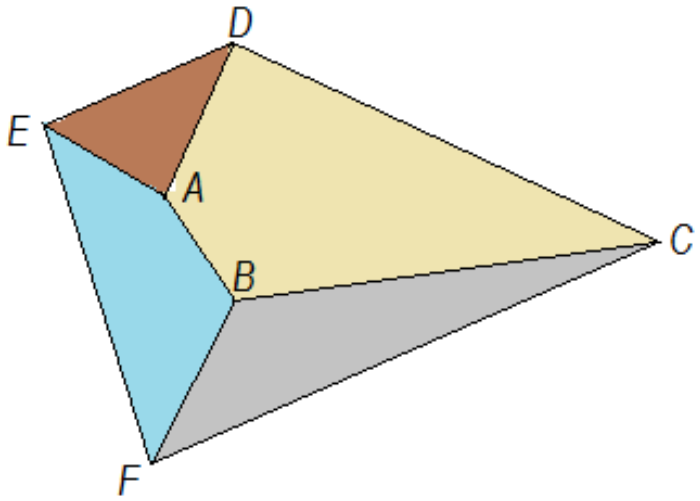
Theme: Volume of a polyhedron

Assumed knowledge: Co-ordinate geometry, position and free vectors, scalar and vector products, vector and parametric equations of lines, vector and Cartesian equations of planes, surface areas and volumes, CAS

A **polyhedron** is a three-dimensional shape.
A **tetrahedron** shown below is an example. It is also called a **triangular pyramid**.
It has **4 vertices**, **4 triangular faces** and **6 edges**.



The following diagram shows another example of polyhedron. It has **2 triangular faces** and **4 quadrilateral faces**, **6 vertices** and **9 edges**.



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Part I (90 minutes plus)

Let $\tilde{i}$, $\tilde{j}$ and $\tilde{k}$ be unit vectors in the direction of positive x, y and z axes respectively. z axis is pointing vertically upwards.

Lengths are measured in arbitrary unit, areas in unit^2 and volume in unit^3.

Three points in space are required to define a three dimensional plane.
Consider the plane (labelled as Π_0) with the three points $T(7,1,5)$, $U(-1,5,4)$ and $V(3,2,6)$ on it.

a. Select two non-parallel vectors on Π_0 and calculate the vector product of your selected vectors.
Find the area enclosed by ΔTUV.

b. Determine a unit vector perpendicular to Π_0.

c. Find a vector equation of Π_0.

d. Show that the Cartesian equation of Π_0 is $5x+12y+8z=87$.

e. Verify that $T(7,1,5)$, $U(-1,5,4)$ and $V(3,2,6)$ are on Π_0.

f. Let L_{tu} be a line through points $T(7,1,5)$ and $U(-1,5,4)$. Find a vector equation of line L_{tu} using t as a parameter. Hence write down a set of parametric equations of line L_{tu}.

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g. Let L_{uv} be a line through points $U(-1,5,4)$ and $V(3,2,6)$.
Write down a set of parametric equations of line L_{uv} in terms of parameter t.
Verify that $U(-1,5,4)$ is the intersection of L_{tu} and L_{uv}.

The Cartesian equations of four planes Π_1, Π_2, Π_3 and Π_4 are
$\Pi_1: 6x+10y+7z=50$, $\Pi_2: 6x+26y+15z=82$, $\Pi_3: 2x+6y+5z=30$, $\Pi_4: 14x+18y+19z=106$

h. Explain why Π_1, Π_2, Π_3 and Π_4 are not parallel to each other.

i. Write down the coordinates of two points of your choice on Π_3. Write down a vector parallel to the line joining your two points in terms of $\tilde{i}$, $\tilde{j}$ and $\tilde{k}$.

Any two of Π_1, Π_2, Π_3 and Π_4 will intersect and the intersection is a line.
For example the intersection of Π_1 and Π_2 is a line. Let the line be L_{12}.
Let L_{23}, L_{34}, L_{13}, L_{14} and L_{24} be the other lines of intersection.
These six lines form the edges of a tetrahedron. The intersection of a pair of these lines is a vertex of the tetrahedron. Your task is to find these vertices.

j. By solving simultaneous equations show that a possible set of parametric equations of L_{12} using t as the parameter is: $x=5-\frac{t}{3}$, $y=2-\frac{t}{2}$ and $z=t$

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k. Show that a possible set of parametric equations of L_{13} using t as the parameter is:

$x = \frac{t}{2}$, $y = 5 - t$ and $z = t$.

l. Show that one of the vertices of the tetrahedron is $(3,-1,6)$. Verify that $(3,-1,6)$ lies on both L_{12} and L_{13}.

m. The parametric equations for each of L_{14}, L_{23}, L_{24} and L_{34} are given below, using t as the parameter.
Note: You are not required to work out these parametric equations.

L_{14}: $x = 5 - 2t$, $y = \frac{4+t}{2}$ and $z = t$

L_{23}: $x = \frac{36-5t}{2}$, $y = -1$ and $z = t$

L_{24}: $x = \frac{40-7t}{8}$, $y = \frac{16-3t}{8}$ and $z = t$

L_{34}: $x = \frac{4-t}{2}$, $y = \frac{13-2t}{3}$ and $z = t$

Show that the other three vertices are $(-2,-1,8)$, $(5,2,0)$ and $(1,3,2)$.

End of Part I

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Part II (50 minutes plus)

Information from **Part I**:

Five planes are $\Pi_0: 5x+12y+8z=87$, $\Pi_1: 6x+10y+7z=50$, $\Pi_2: 6x+26y+15z=82$, $\Pi_3: 2x+6y+5z=30$, $\Pi_4: 14x+18y+19z=106$

The tetrahedron bounded by Π_1, Π_2, Π_3 and Π_4 has vertices $A(-2,-1,8)$, $B(3,-1,6)$, $C(5,2,0)$ and $D(1,3,2)$.

Parametric equations of the six edges forming the tetrahedron are;

$L_{12}: x=5-\frac{t}{3}$, $y=2-\frac{t}{2}$ and $z=t$

$L_{13}: x=\frac{t}{2}$, $y=5-t$ and $z=t$

$L_{14}: x=5-2t$, $y=\frac{4+t}{2}$ and $z=t$

$L_{23}: x=\frac{36-5t}{2}$, $y=-1$ and $z=t$

$L_{24}: x=\frac{40-7t}{8}$, $y=\frac{16-3t}{8}$ and $z=t$

$L_{34}: x=\frac{4-t}{2}$, $y=\frac{13-2t}{3}$ and $z=t$

In **Part II** the task is to find the total surface area and volume of the tetrahedron bounded by Π_1, Π_2, Π_3 and Π_4 using **vector methods**.

a. Calculate the total surface area of the tetrahedron bounded by Π_1, Π_2, Π_3 and Π_4 using **vector methods**.

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b. Calculate the volume of the tetrahedron bounded by Π_1, Π_2, Π_3 and Π_4 using **vector methods**, given volume $= \frac{1}{3} \times base.area \times height$.

c. The plane Π_0 truncates the tetrahedron and only vertex $A(-2,-1,8)$ is above it.
Show that $A(-2,-1,8)$ is the intersection of edges L_{23}, L_{24} and L_{34}.

d. Each of edges L_{23}, L_{24} and L_{34} intersects with plane Π_0. The coordinates of the intersections are $(23,-1,-2)$, $\left(\frac{7}{9},\frac{4}{21},\frac{304}{63}\right)$ and $(7,11,-10)$ respectively.
Show workings in finding the coordinates of these intersections.

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e. Calculate the volume of the *polyhedron* bounded by Π_0, Π_1, Π_2, Π_3 and Π_4.

End of Part II

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